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Michele d'Ottavio, Giuseppe Mantegna, Olivier Polit

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Michele D'Ottavio*, **Giuseppe Mantegna**, **Olivier Polit***

*LEME, UPL, Université Paris Nanterre
50, rue de Sèvres – 92410 Ville d'Avray, France
e-mail: mdottavi@parisnanterre.fr

Summary: This paper presents the extension to geometrical nonlinearities of a variable kinematics modeling approach for composite plates. Geometrical non-linearities are included by referring to von Kàrmàn's finite deflection hypothesis and, assuming the strains remain small, a linear elastic material behavior is retained. The Lagrangian description is adopted for writing the plate motion. A Ritz method is employed to solve the resulting two-dimensional differential equations in weak form within a consistent Newton-Raphson algorithm. First preliminary results verify the proposed implementation.

1. INTRODUCTION

Variable Kinematics (VK) models for composite plates are an effective means to identify the problem-dependent best model in terms of accuracy-to-computational cost ratio through a so-called mixed axiomatic-asymptotic approach [1]. The pioneering VK approach by Carrera (Carrera Unified Formulation, CUF) has been subsequently formally extended by Demasi (Generalized Unified Formulation, GUF) [2] and by D'Ottavio (Sublaminate GUF, SGUF) [3].
Geometrically linear plate models are known to provide accurate results when the out-of-plane deflection is small compared to the thickness of the plate. This is an overly restrictive assumption for realistic applications involving composite plates [4]. In this work, the extension of the SGUF plate models to include von Kàrmàn's non-linear strain is proposed, which allows to consider deflections that are of the same order of magnitude of the plate thickness.

2. FORMULATION

A total Lagrangian description is adopted by referring to Kirchhoff's stress tensor and Green's strain tensor accounting for finite deflections and infinitesimal in-plane displacements:

$$E_{\alpha\beta} = e_{\alpha\beta} + \eta_{\alpha\beta} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) + \frac{1}{2}u_{3,\alpha}u_{3,\beta}; \quad E_{i3} = e_{i3} = \frac{1}{2}\left(u_{i,3} + u_{3,i}\right) \tag{1}$$

where, as usual, $i, j \in \{1, 2, 3\}$ and $\alpha, \beta \in \{1, 2\}$ and Einstein summation convention is adopted. In SGUF, the displacement field is approximated in each sublaminate k as

$$u_i(x_\alpha, z)^k = F_{\alpha_{u_i}}(z)\,\hat{u}^k_{i\alpha_{u_i}}(x_\alpha) \quad \text{with} \;\; \alpha_{u_i} = 0, 1, \ldots N^k_{u_i} \tag{2}$$

The plate equations are derived within the classical virtual work principle and solved by a Ritz method [5]. The non-linear problem is linearized through its incremental form and iteratively solved by a load-driven Newton-Raphson algorithm expressed by tangent and secant matrices:

$$[K_T]\,\{\delta U^{(m)}\} = (\lambda + \Delta\lambda)\{F^R\} \; - \; [K_S]\,\{\delta U^{(m-1)}\} \quad \text{with}\, [K_T] = [[K_0] + [K_u] + [K_\sigma]] \tag{3}$$

where $\Delta\lambda$ defines the incremental load step and δ the increment of the m^{th} iteration, respectively; $[K_0]$ is the incremental stiffness matrix, $[K_u]$ the initial displacement stiffness matrix and $[K_\sigma]$ the initial stress (or geometric) stiffness matrix. First results are shown in Figure 1.

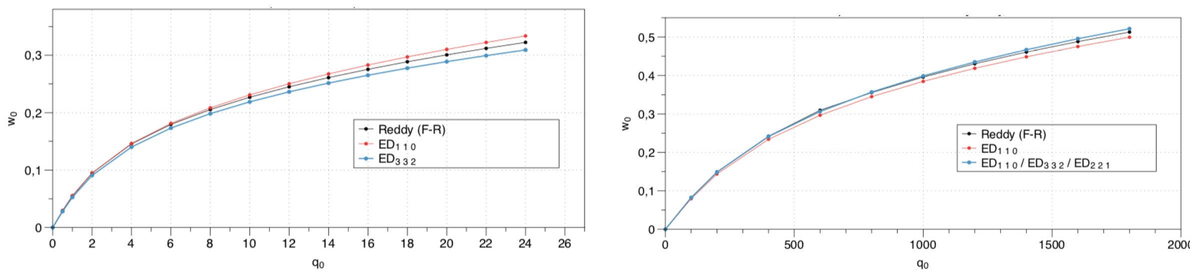

Figure 1. Mid-point deflection *vs* uniform pressure load of square, fully clamped orthotropic plates: one-ply (left) and 6-ply unsymmetric laminate. Reference results from [6].

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