Finite Plate Elements for Viscoelastic Sandwich Structures based on Sublaminate Generalized Unified Formulation

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Algebraic aspects of variable kinematics plate models based on RMVT

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Summary: The modeling of composite plates is discussed based on Reissner Mixed Variational Theorem and variable kinematics assumptions expressed in Generalized Unified Formulation. The analysis of the algebraic properties of the resulting mixed model allows to identify necessary relations between the displacement and the transverse stress unknowns. An attempt is made to find the ``best'' model in view of an accurate transverse stress response.

ABSTRACT

Reissner Mixed Variational Theorem (RMVT) is a variational tool expressly dedicated to composite structures, in which displacements and transverse stresses can be independently assumed [1,2]. It has been used in conjunction with variable kinematics approaches in Unified Formulation to formulate Equivalent Single Layer (ESL) as well as Layer-Wise (LW) models with arbitrary expansion orders for the displacement and stress unknown functions [3—5]. It is well known that any mixed approach of Hellinger-Reissner type requires a careful choice of the static and kinematic field variables in order to avoid an excessively stiff response as well as spurious zero-energy modes and oscillations, such as those depicted in Figure 1. The analysis of the algebraic properties of the mixed matrix allows to identify relations that are required to hold for avoiding such pathological responses [6].

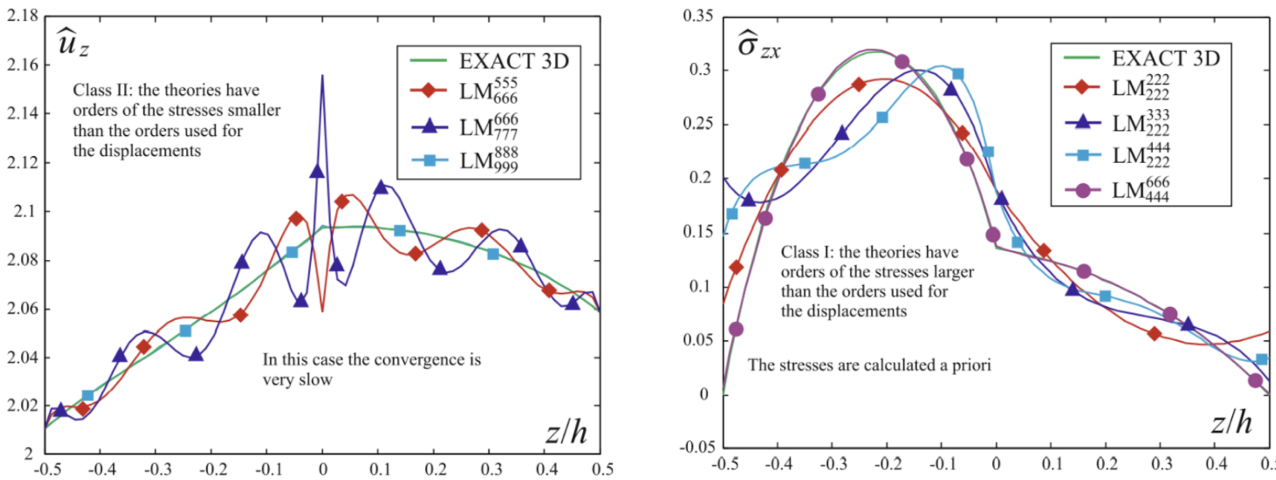

Figure 1. Spurious oscillations of displacements (left) and transverse stresses (right) that may occur in RMVT-based plate models with arbitrary expansion orders for the static and kinematic field variables [results taken from [7].

This work is a first attempt for establishing a general rule for selecting the approximations for displacement and transverse stress variables so to obtain an optimal RMVT-based plate model in the framework of the Generalized Unified Formulation variable kinematics approach [4]. GUF allows to adopt different orders of expansion and descriptions (ESL/LW) for each variable, which shall be regrouped in in-plane displacements, transverse displacement, transverse shear stresses and transverse normal stress. The algebraic system obtained in the framework of Navier-type solutions is considered. Results on homogeneous plates (or, equivalently, full ESL models) are first presented which allow to shed light on the meaning of Fraejis de Veubeke's "Limitation Principle" [8], the possibility of enforcing stress boundary conditions at the plate's top and bottom planes, and on the origins of oscillations and excessive stiffness. The analysis is subsequently extended to composite plates (or, equivalently, to models including a LW description) and a modified version of the original RMVT is discussed in view of an accurate transverse stress response.

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