Structural and acoustic response of sandwich plates by means of sublaminate models with variable kinematics

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Lorenzo Dozio*, **Riccardo Vescovini***, **Michele D'Ottavio**†

* Department of Aerospace Science and Technology
Politecnico di Milano
via La Masa, 34, 20156 Milano, Italy
[lorenzo.dozio, riccardo.vescovini]@polimi.it

† LEME, UPL
Université Paris Nanterre
50 rue de Sèvres, 92410 Ville d'Avray, France
michele.d_ottavio@parisnanterre.fr

Summary

Introduction. Due to their high strength to weight ratio, sandwich panels are adopted as structural members in several engineering applications, such as space launchers and ship hulls. They can be considered a particular case of composite structures, featuring a fundamental pattern of two stiff and thin outer sheets, the skins, enclosing a relatively thick and compliant layer, the core. As opposed to the case of thin-walled panels, a complicating effect of sandwich panels is due to the strong mismatch of geometric and constitutive properties along the thickness, which induces sharp gradients across the thickness direction. For this reason, specific *ad-hoc* structural models are typically required to obtain accurate and reliable predictions of their mechanical behavior. The evaluation of the acoustic performance of sandwich panels can be also crucial in some applications. Indeed, interior noise quality and speech intelligibility has become increasingly important in modern light and fast transportation vehicles, where novel arrangements are designed through the adoption of sandwich configurations to achieve high structural efficiency. In particular, various solutions have been studying in the aerospace industry with the aim of improving the cabin acoustic comfort, especially for VIP aircrafts and helicopters. A common way to reduce the noise radiated by a vibrating fuselage panel is to add viscoelastic materials. The most efficient solution is the so-called constrained-layer damping treatment, where a viscoelastic layer is inserted between the vibrating structure and a constraining rigid layer, thus leading to a sandwich configuration. Another effective approach for noise reduction is to adopt inner fuselage panels, called trim panels, specifically designed for optimizing the acoustic absorption or the transmission loss (TL). Trim panels have typically a classical sandwich multilayer structure with stiff facesheets and a thick low-density core.

Motivation. In both previous approaches, the material and thickness of core and skins of the sandwich panel, the number of layers of external sheets, the topology of the core and the total and relative thickness are, among others, important design parameters affecting the acoustic performances of the structural component. The acoustic design can be further complicated by the adoption of less-conventional and innovative configurations involving multiple cores. Since an extensive experimental campaign is lengthy and costly, suitable numerical simulations can be carried out to guide the design process, i.e., to evaluate the effect of the main parameters and to study the nature and arrangement of layers. However, accurate 2-D modeling of multilayered sandwich panels for vibration and acoustic analysis is challenging, mostly due to their highly heterogeneous anisotropic constitution in the thickness direction and the wide frequency range of interest. In particular, it can be difficult to identify sufficiently simple yet reliable theoretical models without unnecessary complexity.

Mathematical formulation. In this work, a powerful and effective numerical tool for the acoustic analysis of composite panels of finite extent having classical and complex advanced sandwich configurations is presented [1]. Main feature of the proposed formulation is the versatile kinematic representation of the displacement field along the thickness direction, consisting in the representation of the plate by means of sublaminates, i.e. arbitrary groups of adjacent plies composing the panel. Each sublaminate is associated with an independent and arbitrary kinematic description, either equivalent single-layer or layerwise, so that the use of refined high-order theories can be tailored to specific thickness subregions. Accordingly, each component of the 3-D displacement field in the generic ply p of the sublaminate k is postulated in a layerwise (LW) manner as follows:

$$u_\circ(x,y,z_p,t) = F_{\alpha_{u_\circ}}(z_p)\, u^{p,k}_{\circ\alpha_{u_\circ}}(x,y,t) \qquad \alpha_{u_\circ} = 0,\dots,N^k_{u_\circ} \qquad (\circ = x,y,z) \tag{1}$$

where z_p is the local ply-specific thickness coordinate, $F_{\alpha_{u_\circ}}(z_p)$ are thickness functions, $u^{p,k}_{\circ\alpha_{u_\circ}}$ is the kinematic coordinate of the adopted 2-D approximation, $N^k_{u_\circ}$ is the order of expansion taken as a free parameter, and the summation is implied for repeated theory's indexes $\alpha_{u_\circ}$. It is noted that equivalent single-layer (ESL) sublaminate model can be recovered by setting z_p equal to the sublaminate-specific thickness coordinate z_k. The thickness functions are taken as a proper combination of Legendre polynomials so that the continuity between adjacent plies or sublaminates is easily imposed. The equilibrium equations are derived through the Principle of Virtual Displacements (PVD). Once a specific plate theory is postulated through Eq. (1), the corresponding displacement approximation is substituted into the PVD equation so that the original 3-D problem is transformed into a 2-D problem in the $x-y$ plane. The resulting variational form contains 2-D generalized kinematic coordinates, which are further expressed through a Ritz expansion as follows:

$$u^{p,k}_{\circ\alpha_{u_\circ}}(x,y,t) = N_{u_\circ i}(x,y)\, u^{p,k}_{\circ\alpha_{u_\circ} i}(t) \qquad i = 1,\dots,M \qquad (\circ = x,y,z) \tag{2}$$

where $N_{u_\circ i}(x,y)$ form a complete set of admissible functions, which are taken as the product of Legendre polynomials and proper boundary functions after defining the problem in the computational domain (ξ,η), with $\xi,\eta \in [-1,1]$. After substituting Eq. (1) and Eq. (2) into the PVD, the discretized weak form of the dynamic equilibrium equations can be expressed in compact form by means of self-repeating building blocks, denoted as fundamental kernels of the formulation, which are invariant with respect to the number of sublaminates, the typology of the local kinematic description (ESL or LW) and the orders of expansion of each local displacement quantity. Accordingly, the proposed approach allows for the hierarchical generation of plate models with different 2-D kinematic descriptions from the same unified mathematical framework. The expansion and assembly procedure involves four main steps. The first step deals with the expansion of the kernels according to the summation related to the order of the kinematic description postulated in each sublaminate. The second step is the assembly of the ply-contributions in each sublaminate involving a cycling over the index p. All sublaminate contributions are subsequently stacked along the thickness coordinate to account for the continuity of the generic displacement variable at the interfaces between adjacent layers. The sublaminate contributions of different layers are always assembled in a LW manner. The assembly of the sublaminates contributions involves the cycling over the index k. The final step deals with expansion corresponding to the summation related to the Ritz series approximation of the kinematic quantities. Once the final set of equations governing the structural problem is obtained, the acoustic performance of the panel can be evaluated. In particular TL simulations are performed by assuming the plate in an infinite rigid baffle with a negligible air loading and imposing a diffuse field on one side of the panel through a set of incident plane waves of same amplitude and different incidence angle (θ, ϕ). The incident pressure field on the top side of the panel can be expressed as $p = 2e^{-jk\sin\theta(x\cos\phi + y\sin\phi)}$ where $k = \omega/c_0$ is the wavenumber. For each incident wave, the incidence transmission coefficient is computed as

$$\tau(\omega, \theta, \phi) = \frac{2\rho_0 c_0 P(\omega, \theta, \phi)}{S\cos\theta} \tag{3}$$

where S is the panel area and the radiated sound power is evaluated in terms of elementary radiators [2]. Finally, the diffuse transmission loss is computed as $\mathrm{TL}(\omega) = -10\log(\tau_d(\omega))$, where τ_d is the diffuse transmission coefficient derived by integrating the response of all incident plane waves over the incident angle and weighting them with the solid angle to account for the directional distribution.

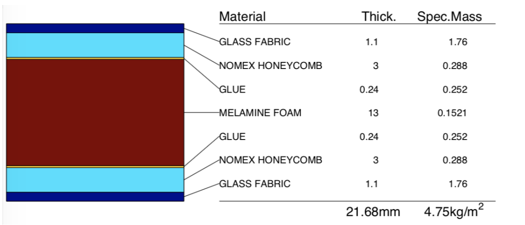

Figure 1: Stacking sequence of a sandwich acoustic trim panel.

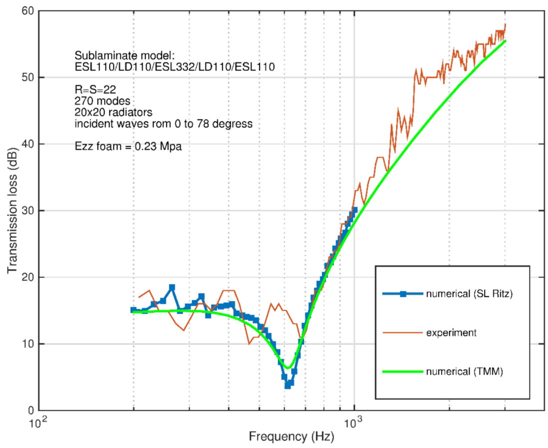

Figure 2: Diffuse transmission loss: comparison between numerical predictions and experimental data.

Illustrative example. Some vibration and TL results on different sandwich plates will be presented and discussed at the Symposium, including also comparison with experimental data, to show the effectiveness of the proposed approach. An illustrative example is here reported in Figure 1, which shows the stacking sequence of an acoustic trim panel of size 0.90×0.90 m^2, thickness 21.7 mm, with clamped boundary conditions and composed of melamine foam placed between Nomex honeycombs and external fiberglass layers. The simulation of the TL and the experimental measurements are compared in Fig. 2. The numerical model is built by properly subdividing the plate into five sublaminates having first-order kinematics in the outer stiffer thickness regions and a higher-order normal and shear deformation theory in the internal soft layer. The high TL is assured thanks to a dilatation effect of the foam from medium frequencies and the static bending stiffness of the honeycombs. The peculiar behavior of a double wall resonance around 600-700 Hz is observed and well predicted by the simulation using a model suitably tuned to give an optimal balance between accuracy and computational cost.

References

[1] D'Ottavio, M.; Dozio, L.; Vescovini, R.; Polit, O.: Bending analysis of composite laminated and sandwich structures using sublaminate variable-kinematic Ritz models. Composite Structures, Vol. 155, pp. 45–62, 2016.

[2] Fahy, F.J.; Gardonio, P.: Sound and Structural Vibration. Elsevier, 2007.