Variable-kinematics approach for an effective modelling of composite structures including multi-field coupling

Michele d'Ottavio

Université Paris Nanterre
200 av. de la République
92001 Nanterre Cedex
www.parisnanterre.fr

École doctorale 139 : Connaissance, langage, modélisation
Laboratoire Energétique Mécanique Electromagnétisme

Membre de l'université Paris Lumières

Dr.-Ing. Michele D'OTTAVIO

Maître de Conférences à l'UFR SITEC de l'Université Paris Nanterre

VOLUME 1 – RAPPORT D'ACTIVITÉ

Variable-kinematics approach for an effective modelling of composite structures including multi-field coupling

Mémoire présenté en vue de l'obtention de

l'Habilitation à Diriger des Recherches

et soutenu publiquement le 3 juin 2022 devant le jury composé de

SALIM BELOUETTAR	Luxembourg Institute of Sci. & Tech.	Président
PHILIPPE BOISSE	INSA Lyon	Examinateur
ERASMO CARRERA	Politecnico di Torino	Rapporteur
BRUNO CASTANIÉ	INSA Toulouse	Rapporteur
ANTÓNIO FERREIRA	Universidade do Porto	Examinateur
OLIVIER POLIT	Université Paris Nanterre	Référent
THOMAS WALLMERSPERGER	TU Dresden	Examinateur

Contenu du mémoire

Organisation du document

Ce mémoire d'habilitation à diriger les recherches comporte 3 parties :

- La première partie est le rapport scientifique qui relate de manière concise mes activités de recherche : cette partie est écrite en anglais.
- La deuxième partie présente mon cursus avec les détails sur ma formation, les activités scientifiques, pédagogiques et administratives, ainsi que mes collaborations internationales. Ma production scientifique, notamment les publications citées dans le rapport scientifique, est présentée Chapitre II.4 avec la convention suivante :

[A]	Article dans une revue internationale à comité de lecture
[B]	Contribution à un ouvrage
[INV]	Conférence invitée
[IC]	Conférence internationale
[NC]	Conférence nationale
[PhD]	Thèse de doctorat
[MT]	Mémoire de fin d'études (Master 2)
[Rep]	Rapport interne

- La troisième et dernière partie est un volume annexe avec une collection choisie de travaux publiés dans des revues internationales ou sous forme de contribution à des ouvrages : le lecteur pourra y trouver des détails et des résultats qui ont été omis du rapport scientifique dans un souci de concision.

  Remarque : Les citations des travaux reportés dans cet annexe sont caractérisées par un astérisque ($\star$), qui est également un lien hypertexte pointant la contribution citée.

Première partie

Variable-kinematics approach for an effective modelling of composite structures including multi-field coupling

I.1

Foreword

The present report aims at describing the research activities that I have been carrying out since my arrival in 2007 at the UFR SITEC in Ville d'Avray, first as a post-doctorate fellow and then, since 2008, as "maître de conférences" (associate professor). These activities are to a great extent in continuity with the studies I could initiate during my PhD at the Institute for Statics and Dynamics of Aerospace Structures (ISD) of the Universität Stuttgart, which focussed on the development of theoretical and computational models for composite structures, including piezoelectric coupling [PhD0].

The competences that I could gather at the ISD allowed me a seamless integration within the activities of the mechanical engineering branch of the LEME (formerly LMpX), the research laboratory associated to the UFR SITEC. I could integrate in a particularly effective manner those activities resting on a computational mechanics approach and concerned with heterogeneous structures and systems: the modelling and simulation of composite structures including multifield coupling (thermo-mechanics, piezoelectricity, electro-magnetics), and the optimisation of laminates and structures with distributed piezoelectric patches. Most of these developments relied on the in-house Finite Element code "EvalEF" initiated by Professor Polit.

My research activities have been developed in particular with the support of 5 PhD theses (one of which is still ongoing) that I had the opportunity to co-direct under the supervision of Professor Polit, as well as of several Master theses carried out under my direct supervision. All activities could also take advantage of numerous collaborations, with my colleagues of the LEME of course, but also with other colleagues in France and abroad.

Note on the document's organisation

The present document conforms the submission requirements for acquiring the French post-doctoral degree accrediting the supervision of doctoral research and is composed of three Parts:

- The following scientific report constitutes Part I.

- Part II is written in French and summarises my academic activities, including teaching activities, details about the PhD theses I could co-supervise, collaborations and scientific output: the list of references pertaining to my own bibliography is to be found in Chapter II.4, organised according to the following labeling:

[A]	Article in an international journal
[B]	Book chapter / contribution
[INV]	Invited lecture
[IC]	International conference
[NC]	National conference
[PhD]	PhD thesis
[MT]	Master thesis
[Rep]	Internal report

- The last Part is an Appendix printed in a separate volume. It contains some selected works published in international journals or as book chapters: details of the developments and results, that have been omitted from this report for the sake of conciseness, can be found in these works.

  Note: The references to works available in this Appendix are highlighted by an asterisk ($\star$), which is also a hyperlink to the cited contribution.

Table of Contents

List of Figures

I.2

Introduction

Due to their excellent structural performances in terms of specific stiffness and strength, composite structures are being increasingly employed for primary load-carrying elements in all weight-sensitive applications, from the aerospace, naval, automotive and railway structures up to rotating components of machineries, such as wind and engine turbine blades and propellers.

The appealing properties of composite structures are attained through their inherent heterogeneity, whose representation leads classically to the definition of three scales: (i) the micro-scale identifies the fibre-matrix heterogeneity within individual plies, (ii) the meso-scale identifies the heterogeneity within the stack of homogenised plies separated by "bi-material interfaces", and (iii) the macro-scale, which resolves the gradients characterising the response of the whole structure.

In order to supply a valuable support for the design of composite structures, computational models should take into account this increased complexity while still satisfying the following fundamental requirements [9]: *reliability* (or accuracy), i.e., to provide results within acceptable errors, and *effectiveness*, i.e., to yield reliable results with the least computational effort.

Current commercial Finite Element (FE) software developers choose to separate the macro- and the meso-scale by providing only two classes of models:

- structural (macro-scale) models: 1D beam and 2D plate/shell elements based on the Classical Laminate Theory (CLT) or First-order Shear Deformation Theory (FSDT), both originally developed for monolithic structures and subsequently augmented to deal with composite stacks [113]
- solid elements that solve the full equations of three-dimensional (3D) continuum elasticity for the local stress analysis at the meso-scale[1]

[1]The use of solid elements for determining the micro-scale response within, e.g., RVE models or direct micro-structure simulations, shall be left out of the present scope that is focused on the structural response.

However, the response of a composite structure does in general not allow such a clear separation between the macro- and the meso-scale. As a result, the underlying hypotheses of the CLT and FSDT may turn out to be excessively constraining. An overwhelming body of literature has been thus dedicated to refined models filling the gap between these "classical" structural models and the full 3D description [28, 114, 128]. The meso-scale gradients that characterise the displacement and stress fields obtained from a 3D solution are exemplarily illustrated in Figure I.2.1 and are customarily referred to as C_z^0-requirements [24, 55].

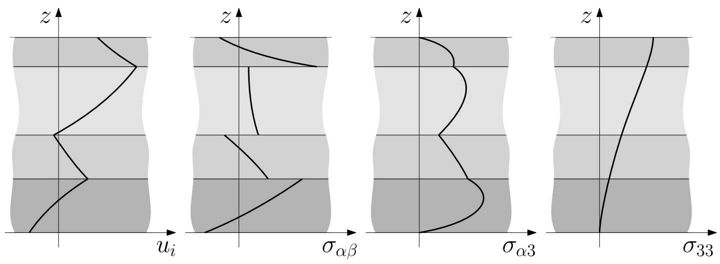


Figure I.2.1 – Schematic of the meso-scale C_z^0 variations of displacement and stress components along the composite stack direction z fulfilling the 3D continuum mechanics equations.

The vast majority of refined models are formulated within the so-called "axiomatic" approach: some *ad hoc* assumptions for the response across the composite stack are introduced into integral expressions typically given by variational principles. Carrying out the integration over the thickness yields the dimensionally reduced model at the macro-scale, i.e., the assumed gradients are homogenised over the meso-scale. Two classes of models can be identified depending on the level of homogenisation [B5⋆]:

Equivalent Single Layer (ESL) models are constructed by integrating over the whole stack, the resulting model is thus independent of the number of layers; the classical CLT and FSDT models pertain obviously to this class, as do successive refinements that may be labelled as High-order Shear Deformation Theories (HSDT) and High-order Shear and Normal Deformation Theories (HSNDT) [1]

Layer-Wise (LW) models carry out the integration over each layer individually so that the resulting model is explicitly represented at the macro-scale [84]

Among the ESL models deserve special mention those that aim at meeting the C_z^0-requirements *a priori*. Such models are often referred to as Zig-Zag Theories (ZZT) in light of their explicit representation of the slope discontinuity that characterise the displacement field at bi-material interfaces [29]. A general procedure for constructing a ZZT upon enriching a generic "basis" HSDT with *warping functions* has been recently proposed by Loredo *et al* [A30⋆], where a discussion has been also provided about the limitations of this kind of ESL approach.

Recent developments of ZZT have been also proposed that refrain from introducing *ad hoc* meso-scale gradients, either resorting to an asymptotic scaling of the potential energy [153, 154] or within a static approach based on the scaling of the external loading with the plate thickness [123].

No unique model has been so far found that is capable of coping with the many parameters that influence the accuracy of the response. In fact, the response depends *to a different extent* on many parameters, such as ply material properties, stacking sequence, structural configuration (geometry) and loading conditions. Multi-field interactions introduce of course additional challenges. As a result, a model that provides reliable results in one case may give unacceptable results in another case, whereas a very detailed model may prove computationally ineffective because it contains an excess of information that is irrelevant for the intended application.

For providing *effective and reliable* analysis tools for a broad range of applications, the quest should therefore be for a series of models rather than for "one model". Such a series of models will be referred to as "variable kinematics models", and its availability brings along several attractive features:

- it allows to compare *a posteriori* the accuracy of one model with respect to another, or, if possible, to the exact 3D solution – this is of particular relevance because axiomatic models do not allow an *a priori* estimate of their pertinence
- physical insight can be gained by allowing to investigate the reasons for the failure of a given model and the success of another
- it allows to establish ranges of validity of a model with respect to certain parameters of the considered problem, i.e., its input data and intended output
- the quantitative estimate of the accuracy with respect to a given response allows to identify a *minimum* computational effort required to achieve the desired accuracy

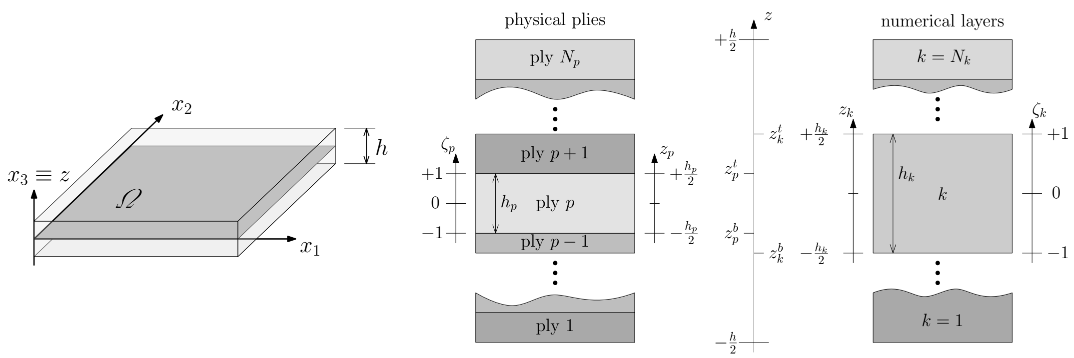
The variable kinematics approach constitutes one of the main objects of the present study and is presented in Chapter I.3; the other research topic addresses the development of reliable and effective solution methods for the model-specific governing equations, which is treated in Chapter I.4. Selected results are reported in Chapter I.5 for illustrating the features of the developed computational models. Conclusions are finally summarised in Chapter I.6 along with an outlook towards possible future research activities.

I.3

Variable Kinematics Models

In the remainder we shall limit the scope to *plate models* for the sake of conciseness, where it is tacitly recognised that the main considerations can be equally applied to beam and shell models, despite curvature effects entail a certain additional complexity [77].

Let the plate occupy, in the Cartesian frame $(O, \vec{x}_1, \vec{x}_2, \vec{x}_3)$, the volume $V = \Omega \times [-\frac{h}{2}, \frac{h}{2}]$, where Ω is the (reference) mid-surface in the (x_1, x_2) plane and h is the plate's thickness measured along the $x_3 \equiv z$ coordinate, see Figure I.3.1a. Further, let the plate be composed of N_p homogeneous plies of uniform thickness $h_p = z_p^t - z_p^b$ over Ω and of at most monoclinic symmetry about the thickness direction z. Numerical layers $k = 1, 2 \ldots N_k$ are introduced such that $h = \sum_{p=1}^{N_p} h_p = \sum_{k=1}^{N_k} h_k$, where $h_k = z_k^t - z_k^b$. Numerical layers can be a fraction of the physical plies ($h_k < h_p$) or, conversely, they may be constituted of a group of $N_p^k \geq 1$ physical plies, as shown in Figure I.3.1b. Whenever $h_p = h_k$ and $N_p = N_k$, plies and layers are synonyms and we shall only refer to the latter. The specific thickness coordinates for the layer k and the ply p are $z_k \in \left[-\frac{h_k}{2}, \frac{h_k}{2}\right]$ and $z_p \in \left[-\frac{h_p}{2}, \frac{h_p}{2}\right]$, respectively; their non-dimensional counterparts are further introduced as $\zeta_k = \frac{2z_k}{h_k}$ and $\zeta_p = \frac{2z_p}{h_p}$, where $\zeta_k, \zeta_p \in [-1, 1]$.



(a) Reference mid-surface Ω (b) Coordinates used across the composite stack

Figure I.3.1 – Coordinates and notation used for describing a composite plate.

Furthermore, let Γ_t and Γ_u be the portions of the plate's boundary $\Gamma \equiv \partial V$ with prescribed surface tractions and prescribed displacement, respectively, where $\Gamma_t \bigcup \Gamma_u = \Gamma$ and $\Gamma_t \bigcap \Gamma_u = \emptyset$. Unless otherwise stated, Greek indices take the values in the set $\{1, 2\}$ whereas Latin indices take their values in $\{1, 2, 3\}$, and Einstein summation convention over repeated indices is adopted.

I.3.1 Variational frameworks

The *ad hoc* assumptions constraining some field variables to describe some given gradients do not allow, in general, to obtain an exact strong-form solution of the elasticity equations. Therefore, the assumptions are introduced into integral statements for obtaining approximated solutions defined in some weighted sense. The dimensional reduction yielding axiomatic structural models is carried out by referring to energy or, more generally, virtual work principles: these allow to derive variationally consistent equations and boundary conditions, and eventually to seek weak-form solutions [150].

Depending on the field variables for which the assumptions are formulated, axiomatic models may be classified as displacement-based (kinematically compatible), stress-based (statically compatible) and mixed models.

Displacement-based (compatibility) approach The most widely adopted approach consists in postulating the distributions of a compatible displacement field along the transverse fiber. The Principle of Virtual Displacements (PVD) reads

$$\iiint_V \delta\epsilon_{ij}\, \tilde{C}_{ijlm}\, \epsilon_{lm}\, \mathrm{d}V = \iiint_V \delta u_i\, b_i\, \mathrm{d}V + \iint_{\Gamma_t} \delta u_i\, \bar{t}_i\, \mathrm{d}\Gamma \tag{3.1}$$

and provides – after carrying out the integrals over the thickness – the variationally consistent equilibrium equations and the traction boundary conditions expressed in terms of stress resultants; the virtual variations are to be taken under the constraints of a *compatible* displacement field satisfying

$$\epsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \qquad \text{and} \qquad u_i = \bar{u}_i \quad \text{at } \Gamma_u \tag{3.2}$$

as well as of the constitutive equations

$$\sigma_{ij} = \tilde{C}_{ijlm}\, \epsilon_{lm} \tag{3.3}$$

where $\tilde{C}_{ijlm}$ is the tensor of the stiffness coefficients of the generalised Hooke law.

The main advantages of the PVD approach lie in its simplicity and versatility, which explain the reason for its broad success. Furthermore, the PVD approach yields governing equations in stiffness form whose solution constitutes a lower bound for the exact displacement field [124]. However, since the equilibrium equations are verified in a weak sense only, displacement-based models do in general not satisfy the stress continuity at bi-material interfaces. Such constraints can be added by augmenting the PVD through opportune Lagrange multipliers, see, e.g., [153, 155].

Stress-based (equilibrium) approach The complementary energy principle, or principle of virtual forces (PVF), allows to introduce approximations for a statically admissible (equilibrated) stress field, and yields compatibility equations in terms of generalised displacements defined as weighted integrals of the local displacements once the constitutive link is exactly verified. In perfect duality with the PVD, the solution of the governing equations derived from the PVF provides an upper bound for the exact displacement field [124]. However, its extension to dynamic and geometrically nonlinear problems has to cope with the difficulty of dealing with generalised displacements [20], which do locally violate the strain-displacement relations. It is interesting to mention the recent development of a "Generalized Reissner" (GR) theory, an entirely consistent extension to composite plates of Reissner's equilibrium approach [115, 116] that includes the full 3D constitutive law [75, 123].

Mixed approaches Mixed variational statements allow to postulate the distribution of variables pertaining to different fields. Hu-Washizu's three-field principle allows introducing arbitrary approximations for all unknown fields u_i, ϵ_{ij} and σ_{ij} and verifies in weak form all equations of 3D elasticity. Fraejis de Veubeke's principle allows independent variations for the strain field and an equilibrated stress field, and entails variationally consistent compatibility and constitutive relations [63]. The most widely adopted mixed principle for formulating axiomatic structural models is however Hellinger-Reissner's (HR) principle [117], in which independent approximations can be introduced for an arbitrary stress field and a *compatible* displacement field, thus yielding variationally consistent equilibrium and constitutive equations. A very systematic presentation of axiomatic plate models formulated in the framework of HR principle is due to Alessandrini *et al.* [4].

Note that the HR principle can be used also without introducing approximations for the displacement field: starting from stress assumptions, the weak form of the constitutive law provides the relations between the generalised displacements and the stress resultants, which finally allows to express the equilibrium equations in terms of generalised displacements [117]. This approach has been specifically used by Pagano for formulating a laminated plate theory for an accurate 3D stress response dedicated to the free-edge effect [101], see also [A12⋆]; this approach has been further extended through the possibility of introducing a hierarchy of approximations for the stress field [21].

Besides their use for building axiomatic models, mixed formulations have gained particular interest in the framework of the FEM as a manner to resolve numerical pathologies that arise within the conventional displacement approach, see, e.g., [89, 129].

RMVT: a mixed approach dedicated to composite structures In the mid-eighties, Reissner proposed a mixed approach specifically thought for formulating models for composite structures [118, 119]. Within this so-called Reissner Mixed Variational Theorem (RMVT), independent approximations are postulated for the displacements and *only the transverse stresses*:

$$\iiint_V \delta\epsilon_{\alpha\beta}\,\frac{\partial\tilde{W}}{\partial\epsilon_{\alpha\beta}} + \delta\epsilon_{i3}\,\sigma_{i3} + \delta\sigma_{i3}\left(\epsilon_{i3} + \frac{\partial\tilde{W}}{\partial\sigma_{i3}}\right)\,\mathrm{d}V = \iiint_V \delta u_i\,b_i\,\mathrm{d}V + \iint_{\Gamma_t} \delta u_i\,\bar{t}_i\,\mathrm{d}\Gamma \tag{3.4a}$$

where the strain field ϵ_{ij} is defined in terms of the assumed displacements by Eq. (3.2) and $\tilde{W} = \tilde{W}(\epsilon_{\alpha\beta}, \sigma_{i3})$ is a "semi-complementary" energy density such that

$$\sigma_{\alpha\beta} = \frac{\partial\tilde{W}}{\partial\epsilon_{\alpha\beta}} = C_{\alpha\beta\gamma\delta}\,\epsilon_{\gamma\delta} + C_{\alpha\beta i3}\,\sigma_{i3} \tag{3.4b}$$

$$\epsilon_{i3} = -\frac{\partial\tilde{W}}{\partial\sigma_{i3}} = C_{i3\alpha\beta}\,\epsilon_{\alpha\beta} + C_{i3j3}\,\sigma_{j3} \tag{3.4c}$$

Starting from assumed *compatible* displacements u_i and *arbitrary* transverse stresses σ_{i3}, and taking the variations under the constraints given by Eq. (3.2) and Eq. (3.4b), RMVT yields as Euler-Lagrange equations the 3 equilibrium equations and the 3 constitutive equations Eq. (3.4c) relating the transverse strains to the in-plane strains and transverse stresses. The coefficients of the mixed form of the generalised Hooke's law read

$$C_{\alpha\beta\gamma\delta} = \tilde{C}_{\alpha\beta\gamma\delta} - \frac{\tilde{C}_{\alpha\beta33}\,\tilde{C}_{33\gamma\delta}}{\tilde{C}_{3333}}; \quad C_{\alpha\beta i3} = \tilde{C}_{\alpha\beta j3}\,(\tilde{C}_{j3i3})^{-1}; \quad C_{i3j3} = (\tilde{C}_{i3j3})^{-1} \tag{3.5}$$

It is worthwhile observing that $C_{\alpha\beta\gamma\delta}$ contains the in-plane *reduced stiffness* coefficients (plane stress), C_{i3j3} the transverse *compliance* coefficients and $C_{\alpha\beta i3} = -C_{i3\alpha\beta}$ the dimensionless Poisson coupling coefficients between in-plane and transverse normal deformations.

Murakami's contributions Since the independently assumed variables of RMVT are those required to be interlaminar continuous at bi-material interfaces, it is straight-forward to define a model that meets *a priori* all C_z^0-requirements; Murakami [96] superposed in fact the simple ZigZag function

$$F_{ZZ}(\zeta_k) = (-1)^k\zeta_k \tag{3.6}$$

to a classical first-order (i.e., C_z^1) in-plane displacement field to introduce the required slope discontinuity at the bi-material interfaces between the layers k and $k+1$. In a subsequent paper Murakami extended RMVT towards imperfect interfaces upon allowing an "interlayer slip" at an arbitrary interface (I) [137]. The virtual internal work of Eq. (3.4a) has been thus augmented to include the contribution of the "sliding" interface:

$$\begin{aligned}\delta W_{int} = \iint_\Omega &\left[\int_h \delta\epsilon_{\alpha\beta}\,\sigma_{\alpha\beta} + \delta\epsilon_{i3}\,\sigma_{i3} + \delta\sigma_{i3}\,(\epsilon_{i3} - \epsilon_{i3})\,\,\mathrm{d}z\right] \\ &+ \delta[\![u_\alpha]\!]^{(I)}\,\sigma_{\alpha3}^{(I)} + \delta\sigma_{\alpha3}^{(I)}\left([\![u_\alpha]\!]^{(I)} - h\,{K_{\alpha\beta}^{(I)}}^{-1}\sigma_{\beta3}^{(I)}\right)\,\mathrm{d}x_1\,\mathrm{d}x_2\end{aligned} \tag{3.7}$$

where the relation is between the displacement jump $[\![u_\alpha]\!]^{(I)}$ and the interlaminar continuous transverse shear stress $\sigma_{\alpha 3}^{(I)}$ is expressed in terms of the *cohesive stiffness* $K_{\alpha\beta}^{(I)}$ of the interface. A non-linear, damageable interface was soon considered within a FE implementation [74].

I.3.2 Unified formulations

The variable kinematics (VK) approach to structural models allows to formulate in a unified manner a number of different models. Implemented in a single computer program, it gives the analyst the possibility to choose the "most appropriate" model for a given application. In the framework of composite structures, it is useful to include ESL and LW descriptions and to select approximations of different order for the gradients along the thickness. Some authors developed VK for displacement-based models to a certain extent (e.g., Matsunaga [92], Soldatos [131] and Reddy [121]), but the most systematic and far-reaching development is certainly due to Carrera and his Unified Formulation (CUF) [33, 36, 39]. CUF has been conceived *ab initio* for constructing RMVT as well as PVD based models, see, e.g., [23, 25]. Demasi subsequently proposed a *formal* generalisation of the original CUF with the so-called Generalised Unified Formulation (GUF) [52, 53]. The development of the Sublaminate Generalised Unified Formulation (SGUF) [A19⋆] is also inscribed within this framework as a *formal* extension of GUF.

I.3.2.1 Carrera Unified Formulation (CUF)

The key-point of CUF relies in the use of a compact index notation for expressing the approximations introduced over the whole composite cross-section (ESL models) or over individual layers (LW models). This index notation permits to formulate the governing equations in terms of theory-independent arrays – the so-called "fundamental nuclei" – and facilitates a straightforward computer implementation: these arrays constitute the theory-invariant kernels from which the governing equations of the structural model are obtained through simple loops that cycle over the desired order of expansion and number of layers.

The displacement and transverse stress variables are expressed in CUF as

$$u_i(x_\alpha, z) = \sum_{\tau=1}^{N} F_\tau(z)\,\hat{u}_{i\tau}(x_\alpha); \quad \sigma_{i3}(x_\alpha, z) = \sum_{\tau=1}^{N} \mathcal{F}_\tau(z)\,\hat{\sigma}_{i3\tau}(x_\alpha) \tag{3.8}$$

where the order N of the approximation is a free-parameter of the model (user-defined input). The layer index k appears explicitly in case of a LW description whereas it is dropped out for an ESL description. Different thickness functions $F_\tau(z), \mathcal{F}_\tau(z)$ can be used for ESL and LW descriptions and the most common choice is the following:

ESL description: a Taylor series expansion $F_\tau(z) = z^\tau$ is used.

Zig-Zag description: Murakami's ZigZag Function defined in Eq. (3.6) is added to the above Taylor expansion.

LW description: a linear Lagrange interpolation for the unknown functions at the layer's top and bottom surfaces is used

$$F_b(\zeta_k) = \frac{1-\zeta_k}{2}; \quad F_t(\zeta_k) = \frac{1+\zeta_k}{2} \tag{3.9a}$$

which is *hierarchically* enhanced up to the order N by approximations defined by

$$F_r(\zeta_k) = P_r(\zeta_k) - P_{r-2}(\zeta_k) \ \text{ with } r = 2, 3 \dots N \tag{3.9b}$$

where $P_n(\zeta)$ is the Legendre polynomial of order n defined as

$$P_0 = 1; \quad P_1(\zeta) = \zeta; \quad P_{s+1}(\zeta) = \frac{(2s+1)\zeta \ P_s(\zeta) - s \ P_{s-1}(\zeta)}{s+1} \tag{3.9c}$$

Note that these approximating functions satisfy the following properties:

$$F_t(1) = F_b(-1) = 1; \quad F_t(-1) = F_b(1) = F_r(\pm 1) = 0 \tag{3.9d}$$

It is worth mentioning that displacement-based CUF models have been also proposed that employ other approximating functions, e.g., trigonometric, exponential, hyperbolic, ... [62].

For further reference, the following remarks about the CUF formalism Eq. (3.8) are pointed out:

1. Eq. (3.8) implies that one single order N is used for *all* variables, which leads to a computer implementation based on "fundamental nuclei" given as 3×3 arrays. These arrays depend on the employed variational framework: for building the stiffness matrix, one and three "fundamental nuclei" are required for PVD and RMVT models, respectively.

   Without modifying the 3×3 form of the "fundamental nuclei", approximations of different orders for specific field variables can be enforced *numerically* by means of penalty factors applied to those terms of the expansion that will be excluded from the final model: this is the underlying technique of the CUF-based Axiomatic/Asymptotic Method (AAM), through which the role of individual contributions of the plate model is assessed with respect to physical parameters of the considered problem (e.g., plate thickness, orthotropy ratio ...) [47, 104, 105]. The AAM provides a sound framework for deriving the "Best Theory Diagrams", which allow to identify the model that yields the most accurate response with the less possible number of unknown functions for a specific application [34, 40].

2. The choice between ESL and LW description is unique for *all* layers of the composite stack and *all* components of the displacement field. A particular CUF implementation with mixed ESL/LW approach has been proposed in [19].

3. The displacement field can be described in ESL or LW manner, while in RMVT-based models the transverse stress field has been always described in LW sense.

Personal contributions within the CUF approach The nowadays well-accepted form of the acronyms introduced for identifying CUF models is recalled in Figure I.3.2a. This variable kinematics approach has constituted the basis of the developments of my PhD thesis [PhD0]: its extension towards piezoelectric materials [A2, A4, A5] and its FE implementation as Abaqus User Element [A3, B1]. The Abaqus User Element has been subsequently employed for the PhD theses of C. Wenzel [PhD2] and T.H.C. Le [PhD4]. I derived the "fundamental nuclei" of the geometric stiffness matrix required for the linearised buckling analysis of plates and shells [A8⋆], which allowed the subsequent investigations on the buckling and wrinkling response of sandwich structures [IC9, IC10⋆, A13⋆, IC14].

I.3.2.2 Generalized Unified Formulation (GUF)

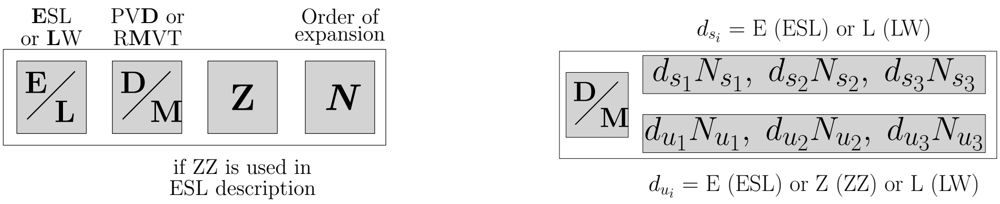
Demasi proposed a *formal* generalisation of CUF by explicitly allowing different approximation orders for each independent field variable [52, 53]. The compact index notation Eq. (3.8) is thus enhanced as follows:

$$u_i(x_\alpha, z) = \sum_{\tau=1}^{N_{u_i}} {}^iF_\tau(z)\, \hat{u}_{i\tau}(x_\alpha); \quad \sigma_{i3}(x_\alpha, z) = \sum_{\tau=1}^{N_{s_i}} {}^i\mathcal{F}_\tau(z)\, \hat{\sigma}_{i3\tau}(x_\alpha) \tag{3.10}$$

where N_{u_i} and N_{s_i} are the expansion orders for the individual displacement and transverse stress variable u_i and σ_{i3}, respectively. The thickness functions used in GUF models are the same of those used in CUF, i.e., ESL models employ Taylor polynomials, LW models the interpolations defined by Legendre polynomials and ZZ models Murakami's ZigZag Function. Also, as in RMVT-based CUF models, the possibility of an ESL description is limited to the displacement field, the transverse stress field having exclusively a LW description. Furthermore, specific ESL, ZZ or LW descriptions could be adopted for each individual displacement component [56], i.e., the thickness functions depend on the index i as indicated in Eq. (3.10): the resulting models have been accordingly named "partial LW" and/or "partial ZZ".

The structure of the acronyms of CUF and GUF models is illustrated in Figure I.3.2, where it is apparent that GUF models require an enhanced notation in order to uniquely identify the model upon specifying *for each variable* the type of description and the expansion order.

Note that the theory-invariant "fundamental nuclei" of GUF models are 1×1 arrays, in contrast to the 3×3 arrays of CUF: PVD and RMVT models require 6 and 14 "fundamental nuclei", respectively, for building the stiffness matrix. The cycling over the expansion order and the assembly of layer-specific contributions follow the very same routines of CUF.



(a) Acronym for CUF models

(b) Acronym for GUF models

Figure I.3.2 – Acronyms for variable kinematics models in CUF (left) and GUF (right)

The possibility of adopting different orders of expansion for the displacement and transverse stress variables stimulated Demasi to conduct a first systematic investigation of the *stability* of RMVT-based models [54]. The usefulness of RMVT-based models depends in fact on the mutual approximations introduced for the kinematic and the static field variables. For instance, the accuracy of transverse stresses obtained *a priori* from RMVT models with ESL displacement field and LW stress field is so insufficient that it is preferable to obtain them in a post-processing step from the integration of the equilibrium equations, just as in displacement-based models [26, 66].

I.3.2.3 Sublaminate Generalized Unified Formulation (SGUF)

The analysis of sandwich panels, for which it is meaningful to adopt different models for the skins and the cores, as well as the literature on delamination simulations, in which the focus is set on particular interfaces between adjacent groups of plies (the so-called “sublaminates”), both suggest that the use of the same model (description, expansion order) throughout the whole composite cross-section may entail unnecessary computational cost. This motivated a further *formal* generalisation of CUF upon explicitly introducing the possibility of applying a generic GUF-type model to individual, arbitrarily defined numerical layers, the “sublaminates”, that are assembled in LW sense assuring interlaminar continuitiy [A19⋆]. The compact notation for the Sublaminate Generalized Unified Formulation (SGUF) reads hence

$$u_i(x_\alpha, z) = \sum_{\tau=0}^{N_{u_i}^k} F_\tau(\zeta)\, \hat{u}_{i\tau}^k(x_\alpha); \quad \sigma_{i3}(x_\alpha, z) = \sum_{\tau=0}^{N_{s_i}^k} F_\tau(\zeta)\, \hat{\sigma}_{i3\tau}^k(x_\alpha) \quad (z \in [z_k^b, z_k^t]) \tag{3.11}$$

where k is the sublaminate index. Figure I.3.3 displays exemplarily how *in each sublaminate*, each individual variable may be ESL, LW or ZZ with any arbitrary order.

In order to facilitate the LW assembly of all sublaminates, independently from the description adopted for the variable in each sublaminate, the thickness functions used in SGUF are *always* those given Eq. (3.9) for a LW description. So, the classical FSDT kinematics is expressed in the sublaminate k in SGUF by the model ED_{110} as follows:

$$\begin{cases} u_\alpha(x_\beta,\zeta_k) = \hat{u}^0_\alpha(x_\beta) + \zeta_k \hat{\varphi}_\alpha(x_\beta) \\ u_3(x_\beta,\zeta_k) = \hat{u}^0_3(x_\beta) \end{cases} \Leftrightarrow \begin{cases} u_\alpha(x_\beta,\zeta_k) = \frac{1-\zeta_k}{2}\hat{u}_{\alpha\,b}(x_\beta) + \frac{1+\zeta_k}{2}\hat{u}_{\alpha\,t}(x_\beta) \\ u_3(x_\beta,\zeta_k) = \hat{u}_{3\,b}(x_\beta) = \hat{u}_{3\,t}(x_\beta) \end{cases} \tag{3.12}$$

where $\hat{u}_{i\,b}$ and $\hat{u}_{i\,t}$ are the displacements at the bottom and top of the layer, respectively[1]. If the sublaminate adopts FSDT in each ply by referring to the model LD_{110}, the non-dimensional coordinate ζ_p is used in Eq. (3.12) instead of ζ_k. So, with reference to the general notation in Eq. (3.11), the non-dimensional coordinate ζ will be either $\zeta = \zeta_k$ or $\zeta = \zeta_p$, depending on whether the variable is described in ESL or LW over the sublaminate. Murakami's ZigZag Function has been also modified in order to be identically nil at the top/bottom surfaces of the sublaminate, so that by virtue of Eq. (3.9d) the assembly can be done straight-forwardly.

The acronym that identifies the model for the whole structure must specify the model used for each sublaminate, which is in turn specified as in GUF by indicating for each variable the adopted description type (E, ZZ, L) and the expansion order, see Figure I.3.2b. Contracted forms are used when, as it is often the case, the same description and expansion order are used for the in-plane displacements or the transverse shear stresses, as in, e.g., $\mathrm{M}^{E2,E2,E1}_{Z2,Z2,E2} \equiv \mathrm{M}^{E2,E1}_{Z2,E2}$. "Backwards compatibility" with CUF model acronyms is assured as, e.g., LM2 $\equiv \mathrm{M}^{L2,L2,L2}_{L2,L2,L2}$, EMZ3 $\equiv \mathrm{M}^{L3,L3,L3}_{Z3,Z3,Z3}$ etc.

SGUF thus affords a very flexible modelling strategy as exemplarily illustrated in Figure I.3.3, in which a sandwich-type section consisting of a thick core, a bottom laminate (4 plies) and a top laminate (12 plies) has been idealised with 3 different SGUF models that adopt a "conventional" three-layers representation but introduce different kinematics for each layer.

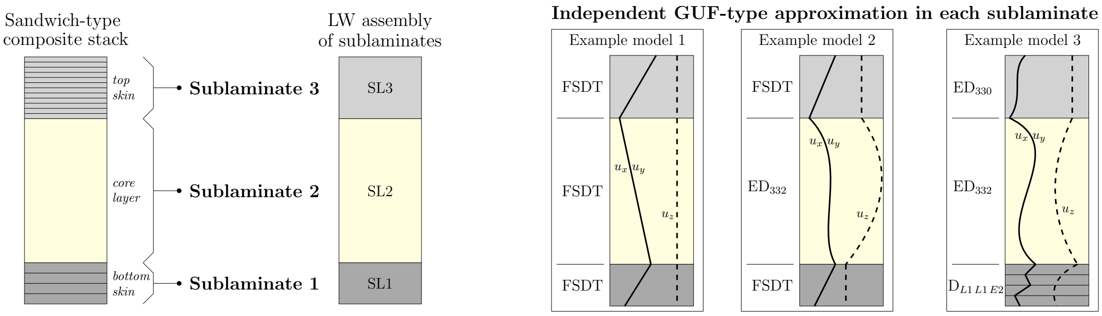


Figure I.3.3 – A composite sandwich section with 3 possible models formulated in SGUF: there are 9 unknown functions for model 1, 15 for model 2 and 27 for model 3. For comparison, CUF model LD1 and GUF model ED_{332} have 54 and 11 unknown functions, respectively.

Model 1 uses the 5-parameter FSDT in each layer and the interlaminar continuity condition $\hat{u}^k_{i\,t} = \hat{u}^{k+1}_{i\,b}$ reduces the number of parameters by 3 for each interface, which yields a model with $5 \times 3 - 6 = 9$ unknown functions;

[1] There is thus a certain analogy with the solid-shell formulations of conventional FEM.

Model 2 enhances Model 1 one upon adopting the 11-parameter kinematics ED_{332} of Lo *et al.* [85] for the thick core, which yields an SGUF model requiring $5 \times 2 + 11 - 6 = 15$ unknown functions;

Model 3 enhances Model 2 by refining the kinematics adopted for the laminated skins: the unknown functions associated to the partially LW kinematics of the bottom skin are $[2(4 \times 2) - (2 \times 3)] + 3 = 13$, those for the HSDT kinematics of the top skin are $2 \times 4 + 1 = 9$, which yields an SGUF model with $13 + 9 + 11 - 6 = 27$ unknown functions.

For comparison, adopting the CUF-type model LD1 for the whole stack (17 plies) requires $6 \times 17 - (3 \times 16) = 54$ unknown functions, while adopting the GUF-type model ED_{332} for the whole stack requires 11 unknown functions. It is thus apparent that the synergy of GUF and a mixed ESL/LW description allows to finely tune the model by selectively enriching the kinematics in specified subdomains (the sublaminates) of the composite stack.

It is worthwhile emphasising the following points:

1. As in GUF, the governing equations are constructed through the assembly of 1×1 "fundamental nuclei". This is entirely managed by opportune pointers, without any need of penalty coefficients for eliminating specific terms.

2. A constant approximation can be used, which is indicated by $N = 0$ in the acronym and is implemented with the thickness function $F_0(\zeta) = P_0$. A variable can also be entirely absent in a sublaminate, which is indicated by a dot $(\cdot)$ in the acronym: this is of relevance for imposing the plane stress condition $\sigma_{33} = 0$ in a RMVT-based model: Murakami's enhanced FSDT model [96] is for instance defined as $\mathrm{M}_{Z1,E0}^{L2,\cdot}$. The possibility of omitting variables from sublaminates is also meaningful in piezoelectric composites whenever the electrical variables are confined within some plies by, e.g., the presence of electrodes.

3. The transverse stress field in RMVT-based models may be described in an ESL sense: this broadens the possibility of adopting "matching" approximations for displacement and stress variables. The possibility is also given to enforce *a priori* boundary conditions for the transverse stress field at the plate's top/bottom surfaces.

I.3.3 Multi-field coupling

Some considerations are next proposed for spanning the frame of the application of the axiomatic approach to problems involving multi-field coupling. The multi-field problems of concern pertain to the class of so-called "volume-coupled" problems, in which different physical fields coexist and interact within the same material region through the constitutive relations

[157]. The inclusion of material layers with multi-field interactions naturally leads to the definition of *smart laminates*, which have attracted considerable interest in view of an extension of the structural functionalities beyond the passive load-bearing capability, for implementing, e.g., adaptive structural morphing, multifunctional sensing, energy harvesting, active vibration control . . . [46]. Appropriate models are thus required for properly resolving the interacting fields and their mutual influence, and the variable kinematics approach provides with mixed variational statements an extremely useful and flexible framework [A1, 12, 32, 38, A4].

In this work the focus is restricted to the *weak* thermo-mechanical coupling and the *strong* piezoelectric coupling; furthermore only the "generalised displacement" approach will be considered for the subsequent numerical implementation, in which the conventional displacement-based principle is extended to include temperature and electrostatic potential variables ("electric enthalpy" formulation [6]). The pyroelectric coupling shall be neglected also.

Thermo-mechanics In the *weak* thermo-mechanical coupling, the temperature field is imposed and is not modified by the mechanical response. Simple distributions are most often employed, in which the temperature profile is uniform or linearly varying across the thickness of the composite stack; more complex temperature profiles are obtained upon solving the heat conduction problem, which may yield nonlinear and LW profiles [27]: the index notation of CUF can be conveniently employed to describe the temperature profile, also, and the variable kinematics approach allows to adapt the description of the mechanical response to the adopted temperature profile. It is important to stress the quasi-isotropic nature of the thermal expansion coefficients, which therefore induces transverse normal strains of the same order of magnitude as the in-plane strains [30], even if a simple uniform temperature profile is used.

Piezoelectricity The *strong* piezoelectric coupling requires particular care in the choice of assumptions for the kinematics and the electrostatic potential in order to enable the study of actuator and sensor configurations [120, 126]. Furthermore, contrary to the quasi-isotropic thermal deformation, the piezoelectric coupling depends on the actuation mode: our contribution [A7⋆] aimed at identifying the dedicated assumptions to be introduced for resolving the piezoelectric transverse extension mode (31-mode) and transverse shear mode (15-mode) for actuator and sensor applications. Finally, the piezoelectric response is usually confined within electrodes, modelled as equipotential surfaces at which electrical conditions are specified: a LW description of the electrical variables is hence recommended for granting a direct access to the value of the electrostatic potential at the bounding electrodes of a piezoelectric layer.

I.4

Solution of the 2D problems

Introducing the axiomatic model into the variational formulations and carrying out explicitly all the integrals over the thickness of the plate yields the 2D governing equations in weak form. Among the several possible solution methods for these 2D problems, the attention has been restricted to the following three approaches, which can be hierarchically classified with respect to their range of application and accuracy:

Navier-type solution: it allows to solve all Euler-Lagrange equations of the variational statement in strong form for a particular, restricted class of problems: this solution is thus particularly useful for assessing the model with respect to the *interior solution* of the problem, i.e., in absence of edge effects.

Ritz method: it finds a weak form solution of the variational problem by means of shape functions defined over the whole domain Ω (global approximation): its range of applicability is restricted by the limitations dictated by the implemented features (geometry, analysis type, boundary conditions and loading ...) and by numerical ill-conditioning that may arise if high-order shape functions are used, e.g., for resolving steep gradients.

Finite Element Method: FEM can be interpreted as a piecewise Ritz-type solution, whose solution is defined over small subdomains Ω_e by *interpolating* shape functions, which allow a straight-forward assembly of the elementary domains to form the global response. The subdivision into subdomains allows to model arbitrary geometries and to adopt low-order shape functions, thus reducing ill-conditioning problems that may affect the Ritz method.

I.4.1 Navier solution

For the strong-form Navier-type solution to exist, the problem is restricted to a plate whose constituting materials have their principal axes aligned with the reference frame used for the analysis. For a time-harmonic response of circular eigenfrequency ω, the Navier solution has the general form:

$$\{\hat{u}_{1\,\tau}(x_1, x_2; t),\ \hat{\sigma}_{13\,\tau}(x_1, x_2; t)\} = \{U_{1\,\tau},\ S_{13\,\tau}\}\ \cos\frac{\pi x_1}{L_x}\ \sin\frac{\pi x_2}{L_y}\ e^{\mathrm{i}\,\omega\,t} \tag{4.1a}$$

$$\{\hat{u}_{2\,\tau}(x_1, x_2; t),\ \hat{\sigma}_{23\,\tau}(x_1, x_2; t)\} = \{U_{2\,\tau},\ S_{23\,\tau}\}\ \sin\frac{\pi x_1}{L_x}\ \cos\frac{\pi x_2}{L_y}\ e^{\mathrm{i}\,\omega\,t} \tag{4.1b}$$

$$\{\hat{u}_{3\,\tau}(x_1, x_2; t),\ \hat{\sigma}_{33\,\tau}(x_1, x_2; t)\} = \{U_{3\,\tau},\ S_{33\,\tau}\}\ \sin\frac{\pi x_1}{L_x}\ \sin\frac{\pi x_2}{L_y}\ e^{\mathrm{i}\,\omega\,t} \tag{4.1c}$$

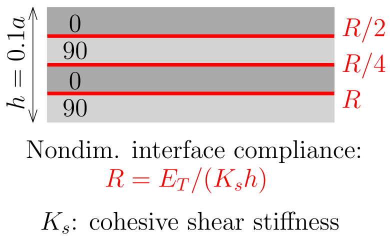
This solution defines a periodic response characterised by the half-wavelengths L_x and L_y along the 2 directions $\vec{x}_1, \vec{x}_2$ of an *infinite plate*: whenever edge effects can be neglected, this approach is very efficient for computing short wavelength responses [A13⋆, A21⋆].

Upon defining $L_x = a/m$ and $L_x = b/n$ with $m, n = 1, 2, 3\ldots$, the above periodic solution is associated to a *finite* plate of rectangular planform $\Omega = [0, a] \times [0, b]$ with "simply-support" boundary conditions identical to those used for constructing exact elasticity solutions [100]. Navier solution can be then inscribed within a Fourier series development for representing bending loads of arbitrary shape, see, e.g., [48]. Cylindrical bending problems related to *wide plates* are solved by simply setting $n = 0$ (i.e., $L_y \to \infty$; if *narrow plates*, or equivalently plane beams, are to be considered, the position $n = 0$ must be complemented by the modification of the material constants so to enforce the *plane stress* condition $\sigma_{22} = 0$. The solution Eq. (4.1) can be also applied to shells of constant curvature [A2, A8⋆].

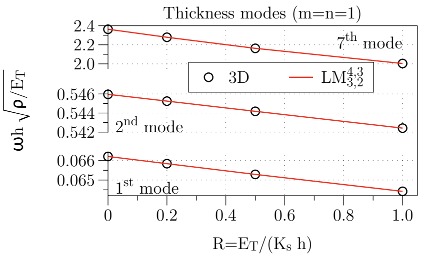
By virtue of the solution in strong form of the gradients over Ω, Navier solution is conveniently used for testing and assessing axiomatic models [A19⋆]. The RMVT approach for linear elastic ("imperfect") interfaces allowing for interlaminar slip has been implemented for the variable kinematics SGUF models in [MT4]. As an example, the effect of imperfect interfaces on the vibration response of a square laminate with three imperfect interfaces of different cohesive stiffness is shown in Figure I.4.1 [IC17]. Besides the perfect agreement of the present high-order GUF model $\mathrm{LM}_{3,3,2}^{4,4,3}$ with the reference solution, it is highlighted that the CUF model LM2 entails a spurious kinematic mode due to the mismatching approximations used for the transverse shear stresses and the interfacial *discontinuous* in-plane displacements. This result confirms the importance of allowing different expansion orders for the static and kinematic variables in mixed models.

I.4.2 Ritz approach

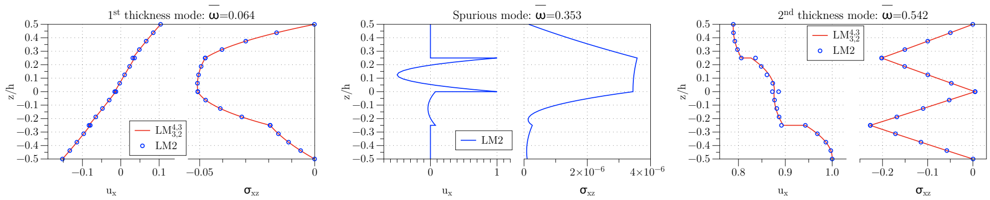
The method proposed by Ritz is a weighted residual method that generalises the solution method formulated by Rayleigh for solving vibration problems [78]. The weak form solution for the generic field variable $\hat{\mathcal{U}}(x_1, x_2) \in \{\hat{u}_{i\,\tau}, \hat{\sigma}_{i3\,\tau}\}$ of the 2D problem is sought within the space spanned by global approximation trial functions $N(x_1, x_2)$. These are required to exactly verify the essential boundary conditions associated to the variational formulation. As



(a) Definition of the imperfect interfaces



(b) Validation against 3D solution



(c) Distributions $u_1(z)$, $\sigma_{13}(z)$ for the 1^{st} and 2^{nd} modes and the spurious mode introduced by LM2

Figure I.4.1 – Navier solution for the free-vibration response of a square cross-ply plate with imperfect interfaces of non-uniform stiffness: (a) Description and reference 3D solution available in [43] – (b) The eigenfrequencies of the 1^{st}, 2^{nd} and 7^{th} thickness modes (first flexural mode: $m = n = 1$) are lowered as the interface compliance increases – (c) Through-thickness modal shapes for the 1^{st} and 2^{nd} modes: CUF model LM2 introduces a spurious kinematic mode.

pointed out in Section I.3.1, for PVD and RMVT formulations these concern only the kinematic variables, i.e., the generalised displacements $\hat{u}_{i\,\tau}(x_1, x_2)$ at the plate boundary.

The Ritz method has been implemented for plates with rectangular or skew planforms with four straight edges; upon introducing the natural coordinates ξ_1, ξ_2 for mapping the physical domain Ω to the computational domain $\Omega_\square : [-1,1] \times [-1,1]$, the Ritz solution is expressed as

$$\hat{\mathcal{U}}(\xi_1, \xi_2) = \sum_{i=1}^{M} N_i(\xi_1, \xi_2) U_i = \sum_{m=1}^{R} \sum_{n=1}^{S} \phi_m(\xi_1)\, \psi_n(\xi_2)\, U_i \quad (i = S(m-1) + n) \tag{4.2a}$$

where the trial functions are defined as products of the 1D "beam" functions

$$\phi_m(\xi_1) = p_m(\xi_1)\, b_{\mathcal{U}}(\xi_1); \qquad \psi_n(\xi_2) = p_n(\xi_2)\, b_{\mathcal{U}}(\xi_2) \tag{4.2b}$$

in which $p(\xi)$ are *orthogonal* polynomials and $b_{\mathcal{U}}$ are boundary functions that satisfy the boundary conditions specified for the generic variable $\mathcal{U}$ at the four edges $\xi_\alpha = \pm 1$ of the plate:

$$b_{\mathcal{U}}(\xi_\alpha) = (1+\xi_\alpha)^{e_{1\mathcal{U}}}(1-\xi_\alpha)^{e_{2\mathcal{U}}} \quad \text{with} \begin{cases} e_{1\mathcal{U}}, e_{2\mathcal{U}} = 1 & \text{if } \mathcal{U}(\xi_\alpha = \pm 1) = 0 \\ e_{1\mathcal{U}}, e_{2\mathcal{U}} = 0 & \text{if } \mathcal{U}(\xi_\alpha = \pm 1) \text{ free} \end{cases} \tag{4.2c}$$

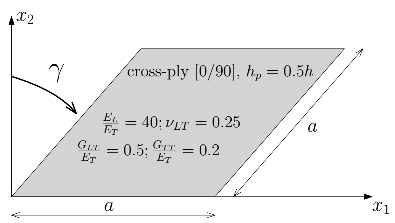
This solution has been applied to CUF models [58] and subsequently extended to SGUF models [A20⋆]. Following the compact notation of Unified Formulations, the governing equations are written in terms of "fundamental nuclei" of the Ritz solution, which are expanded and assembled upon cycling over the indices of the through-thickness model (in SGUF, $\tau \in [0, N_{\mathcal{U}}^{k}]$ for the k$^{\text{th}}$sublaminate) and those of the in-plane solution ($i \in [1, M = R\,S]$). The *kernel integrals* of the Ritz approximation are evaluated in an exact, analytical manner, which enhances substantially the computational efficiency. The thorough numerical investigation conducted in [A29⋆] has demonstrated the superiority of Legendre and Chebyshev polynomials over trial functions defined by the trigonometric functions given in Eq. (4.1): even after enhancing the basis upon including the linear term required for representing a state of uniform strain, the trigonometric functions in Eq. (4.1) inherently define some *natural* boundary conditions that, in general, do *not* correspond to those of the problem. Furthermore, Legendre polynomials outperform Chebyshev polynomials in terms of computational time because of the enhanced sparsity of the stiffness matrix. Finally, no ill-conditioning has been found to affect the stability of the solution even if as high as 250 "beam" functions are used. The proposed Ritz solution of SGUF models has been recently extended to closed and open shells with constant curvature for PVD-based [65] and RMVT-based models [MT8]

Viscoelastic damping Since the Ritz method is particularly suited for vibration analysis, dedicated procedures have been implemented allowing the computation of frequency-response functions, the extraction of modal-based reduced models and the post-processing of acoustic indicators [IC31]. The possibility of dealing with damping viscoelastic materials has been taken into account also: the Complex Modulus Approach has been invoked for modelling frequency-independent as well as frequency-dependent viscoelastic materials in the frequency domain. The dependency of the material moduli on the frequency can be represented in terms of a General Maxwell Model (GMM) as proposed in [79] or a 4-parameters Fractional Derivative Zener model (FDZ) [112]. Several algorithms have been implemented for extracting the eigenfrequencies and associated modes of a frequency-dependent, complex stiffness matrix, such as the Iterative Complex Eigensolver (ICE) and the Modal Strain Energy (MSE) approach along with its Iterative (IMSE) and Modified (MMSE) versions; a comprehensive discussion about modal projection algorithms can be found in the paper by Rouleau *et al.* [122]. All details about the current implementation of the viscoelastic behaviour within the SGUF-Ritz tool can be found in our paper [A33⋆].

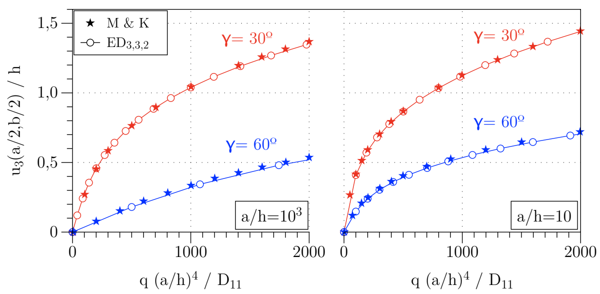
The computational effectiveness of the proposed SGUF-Ritz tool has motivated its use within innovative optimisation algorithms for the design of composite panels with optimal noise attenuation [IC33]. This study is being carried out at IST Lisbon in the framework of a collaboration established for the PhD thesis of G. Di Cara, part of which is dedicated to the analysis of viscoelastic damping.

Extension to multi-field problems The SGUF-Ritz tool has been extended to thermal buckling problems under temperature-induced in-plane loads [A26], and the possibility of including piezoelectric layers has been presented in [A27⋆]. The variable kinematics approach permits to resolve the field interactions up to the desired level of accuracy. The implementation, based on the "generalised displacement" formulation, considers the most frequently employed 31-mode, typical of piezoelectric layers that are bonded at the outer surfaces of a "basis structure". A LW description of the electrical variables is retained, which allows to prescribe the values of the electrostatic potential at the electroded interfaces whenever required. Since the difference between open-circuit and short-circuit conditions allows to quantify the effective electromechanical coupling [14], it is important to represent the equipotential condition at electroded interfaces: to this aim, a one-term, constant shape function has been adopted for the electrostatic potential variable at the electroded surface instead of the series expansion Eq. (4.2a).

Extension to geometrical nonlinearity Due to the computational efficiency of the implemented Ritz method, this framework has been used to host a first extension of the SGUF approach towards geometrical nonlinearity within von Kármán approximation. Great part of this work has been accomplished by G. Mantegna during his Master thesis [MT5] and it has been first presented in [IC30]. A consistent Newton-Raphson linearisation has been implemented within a conventional load-controlled incrementation scheme as well as several path-following schemes with arc-length constraints [22]. Contrary to the linear analysis, ill-conditioning problems have been encountered when polynomial orders $R, S > 12$ are employed for the Ritz approximation, despite an exact analytical evaluation of the kernel integrals of the non-linear matrices. Nevertheless, the method can be applied to relatively smooth problems, for which a converged Ritz approximation can be found with $R, S \leq 12$. An example of non-linear bending response is illustrated in Figure I.4.2 for a fully-clamped laminated skew plate [IC30].



(a) Definition of the testcase [88]



(b) Nonlinear bending response $(R = S = 6)$

Figure I.4.2 – Nonlinear bending of a skew laminate under uniform pressure q. M&K results were obtained with a Differential Quadrature Method and FSDT [88].

Extension to mixed RMVT models This work, realised by P. A. Esposito within his Master thesis [MT8], has allowed to perform, for the first time, some stability analyses of *mixed* RMVT models in conjunction with Ritz method, see e.g., Figure I.4.3. Let $M_U = R_U S_U$ and $M_S = R_S S_S$ denote the size of the polynomial space spanned by the Ritz approximation for

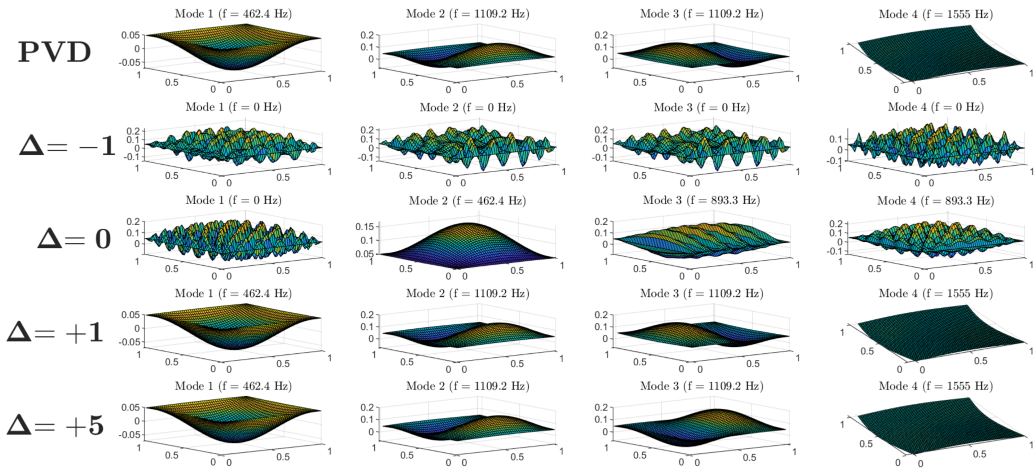


Figure I.4.3 – First 4 flexural modes of a simply-supported isotropic square plate: comparison of PVD results (ED$_{1,0}$, $R_U = S_U = 15$) against RMVT results obtained with EM$^{0,\cdot}_{1,0}$ and different expansion orders for the stress variables $R_S = S_S$ defined by $\Delta = R_S - R_U$. Results extracted from [MT8]

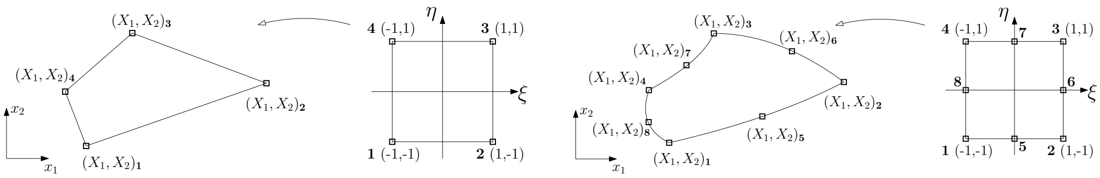
the displacement and transverse stress variables, respectively; the main results are

1. It has been numerically confirmed that no boundary conditions are required for the trial functions of the transverse stress field.
2. Spurious zero-energy modes appear if $M_U \geq M_S$, more specifically if $R_U \geq R_S$ and $S_U \geq S_S$.
3. Convergence to the correct response is obtained by setting $M_S \geq M_U + 1$, more specifically $R_S \geq R_U + 1$ and $S_S \geq S_U + 1$.
4. Considering the additional computational time wasted by unnecessary transverse stress DOF, it is recommended to select $R_S = R_U + 1$ and $S_S = S_U + 1$.

I.4.3 FEM implementation

The developments within the Finite Element Method presented in this Section strive for the following two objectives:

1. to enhance the *reliability* of plate/shell FE for variable kinematics models formulated in CUF and SGUF: develop plate/shell FE that are free of numerical pathologies for ensuring convergence with appropriate rates towards the reference solutions obtained, e.g., by the previously discussed Navier or Ritz solutions;



(a) Four-node (Lagrange) plate FE (b) Eight-node (Serendipity) plate FE

Figure I.4.4 – Geometries of the 4- and 8-node plate FE and their corresponding representation in the parent domain: the map is defined by the Jacobian transformation $\boldsymbol{J} : (\xi, \eta) \rightarrow (x_1, x_2)$.

2. to enhance the *effectiveness* of the FEM towards challenging applications typical of advanced composite structures: develop specific plate/shell models for multi-field coupling along with suitable FE approximations that allow to minimise the computational cost without affecting the accuracy of the desired response.

I.4.3.1 Development of robust FE for variable kinematics plate models

All displacement-based and mixed CUF plate models have been implemented as Abaqus User Elements for exploiting some useful pre-processing functionalities, such as the mesh generation, as well as the highly optimised memory allocation and computational procedures of a commercial software. Furthermore, it allows to exhibit the compatibility of the variable kinematics approach with commercial FE packages.

Figure I.4.4 displays the general quadrilateral four-node FE (bi-linear Lagrange) and the eight-node FE (quadratic Serendipity) that have been implemented for flat plate elements formulated within PVD and RMVT. The main motivations driving the implementation of the quadratic element have been to improve the discretisation of curved boundaries and to enhance the convergence rate; its higher accuracy for evaluating the transverse stress fields through the integration of the equilibrium equations has not been taken into consideration because the focus has been strenuously set on the capability of refined models to obtain accurate transverse stress fields directly from the computation. As a starting point, the same C^0 isoparametric interpolations are used for the generalised displacements $\hat{u}_{i\,\tau}$ and the transverse stress variables $\hat{\sigma}_{i3\,\tau}$ [37, PhD0]. Modified interpolation schemes have been also considered for PVD and RMVT based elements as discussed in the following.

Displacement-based FEM

Isoparametric displacement-based elements for shear-deformable models are known to suffer the transverse shear locking pathology: for thin plates the convergence rate with *practically relevant mesh densities* is dramatically reduced because the constraint of zero transverse shear deformation implies a spurious constraint on the bending deformation. This spurious constraint is less important as the polynomial order of the isoparametric FE interpolation gets high: for sufficiently rich interpolation schemes, such as those adopted in the isogeometric FEM, the transverse shear locking does not manifest [59, 81].

The study carried out in [A3] demonstrated that all displacement-based isoparametric CUF plate elements display the same quantitive convergence rate reduction, *irrespective of the order of the assumption for the transverse shear deformation introduced by the model*: HSDT suffer transverse shear locking in *exactly* the same manner as FSDT; in other words, the shear locking pathology is introduced by the FE interpolation scheme and is independent from the order of the plate/shell theory. In fact, the transverse shear strain field of a generic HSDT can be expressed as

$$\boldsymbol{\gamma}(x_1, x_2, z) = \boldsymbol{\gamma}^{\mathbf{0}}(x_1, x_2) + \boldsymbol{\gamma}^{\boldsymbol{h}}(x_1, x_2, z) \tag{4.3}$$

in which the $z-$constant contribution $\boldsymbol{\gamma}^{\mathbf{0}}$ is that of FSDT. Since the remaining contribution $\boldsymbol{\gamma}^{\boldsymbol{h}}$ depends on z, it goes naturally to zero as the plate becomes thin, and so does the spurious constraint associated to the high-order shear terms. Therefore, all remedies that have been developed for alleviating the shear locking pathology affecting the classical FSDT model can be carried over to high-order shear deformation theories. These can be classified in reduced integration techniques and so-called B-bar methods [70], which introduce a substitute interpolation of the transverse shear strain field [8, 10, 71, 87, 102, 132]. Both class of remedies can be applied to the whole transverse shear strain field $\boldsymbol{\gamma}(x_1, x_2, z)$ [35, 37] or to *only* the $z-$constant contribution $\boldsymbol{\gamma}^{\mathbf{0}}(x_1, x_2)$. We followed the latter possibility and adopted the QC4 and CL8 interpolation schemes for the four- and eight-node elements, respectively, which were developed by Polit [107–109] following the field-consistency paradigm [110] and the original contributions of Prathap and co-workers [111, 132].

The specific interpolation for $\boldsymbol{\gamma}^{\mathbf{0}}(x_1, x_2)$, illustrated in Figure I.4.5 for the 4- and 8-node elements, is introduced in the natural coordinates of the parent domain, the mapping with the physical domain being accounted for by the Jacobean transformation (see Figure I.4.4). As a result, the QC4 and CL8 elements show a remarkably low sensitivity to element distortion and can be reliably employed even in highly irregular meshes. This is of practical relevance for it allows to efficiently discretise irregular geometries, e.g., plates with holes. A recent collaboration with Cinefra and Carrera allowed to extend their CUF-based MITC4 and MITC9 elements towards distorted plate element geometries [A32]. Furthermore, all contributions to the element's stiffness matrix are evaluated by means of Gaussian quadrature schemes of appropriate order, i.e., all energy contributions are exactly evaluated: a full-rank stiffness matrix is thus obtained with only 6 zero eigenvalues and no spurious zero-energy modes. The implementation of the QC4 and CL8 elements into the Abaqus UEL has been accomplished by T.H.C. Le within her PhD thesis [PhD4, A24⋆, B6⋆].

RMVT-based elements

RMVT-based 4- and 8-node elements have been implemented in parallel to the displacement-based ones discussed above. We shall next refer to *mixed* and *hybrid* FEs according to the

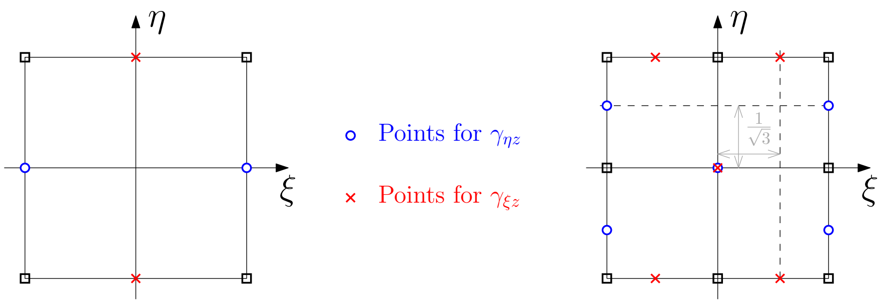


Figure I.4.5 – Tying points used for defining the substitute interpolation of the transverse shear strains: QC4 (left) and CL8 (right)

following definitions: *mixed* FEs are directly obtained upon discretising the RMVT and have generalised displacements and stresses as nodal DOF; *hybrid* elements are obtained from the mixed FE arrays upon statically condensing out the "stress parameters" at element level and have only generalised displacements as nodal DOF, see also [37].

Hybrid elements Isoparametric *hybrid* elements are shown to suffer the very same locking pathology of conventional PVD-based elements. The QC4 and CL8 interpolations have been thus employed for the $z-$constant transverse shear *strain* field, which allows to recover the expected convergence rate irrespective of the element thickness and to obtain a remarkable robustness against distorted element geometries. On the other hand, the use of a dedicated interpolation for the transverse shear stress field is a well known remedy to construct locking-free *hybrid* elements [7, 76]. So, as an obvious choice, the interpolation schemes illustrated in Figure I.4.5 have been used to interpolate the transverse shear "stress parameters", while the isoparametric scheme is retained for the displacement and transverse normal stress variables [B6⋆]. These so-called QC4S and CL8S *hybrid* elements are indeed locking-free but they show, however, poorer performances compared to their QC4 and CL8 counterparts in two respects: (i) the stiffness matrix of CL8S is affected by spurious zero-energy modes; (ii) the QC4S and CL8S display a much larger sensitivity with respect to mesh distortion.

Mixed elements As far as *mixed* RMVT elements are concerned, isoparametric interpolation does not introduce any transverse shear locking pathology. The numerical evidence for this behaviour provided in our studies [PhD0, PhD4, A24⋆, B6⋆] has been recently confirmed by Demasi and Santarpia on the basis of the functional reconstitution technique [57]. The use of QC4 and CL8 interpolations is nevertheless recommended for the transverse shear strains in order to enhance the robustness of these *mixed* elements with respect to distorted meshes.

FEM implementation of SGUF plate models

The sublaminate variable kinematics approach SGUF has been also dedicated an entirely new, in-house FE code development for circumventing certain limitations of the commercial tool as well as for easing the portability and the development of dedicated interfaces. This work is being accomplished by G. Di Cara in the framework of his PhD thesis, in continuity with his Master thesis [MT6]. PVD and RMVT-based models are included. All thickness integrals are represented analytically and only their numerical values are evaluated at runtime. The general 4- and 8-node displacement-based, hybrid and mixed plate elements developed by T.C.H. Le [PhD4] are implemented into a Fortran code based on a sparse array representation allowing the use of computationally efficient routines: the direct PARDISO solver for linear systems and the iterative ARPACK solver for generalised eigenvalue problems. First results confirming the reliability of the implemented FEM and their application to bending problems of sandwich plates have been presented in [IC32]. The code can deal with complex arrays, which allows to model viscoelastic materials with frequency-independent [NC6] as well as frequency-dependent [IC34, MT9] moduli, according to the same representations and procedures already available in the SGUF-Ritz tool.

I.4.3.2 Development of effective FE-based computational tools for advanced composite structures

Several developments aiming at enhancing the effectiveness of FEM have been carried out into an in-house software instead of Abaqus. This allows to provide prototypical implementations without the overhead of coping with predefined programming interfaces. The Fortran code EvalEF, initiated by O. Polit and further developed by P. Vidal, offers a broad library of displacement-based beam, plate and shell elements that discretise classical models (Euler-Bernoulli and Timoshenko beams, Kirchhoff-Love/Koiter and Reissner-Mindlin/Naghdi plates/shells) as well as several high-order models pertaining to the Sinus-family [146–148, A15]. The following extensions of the classical FEM have been addressed:

1. The possibility of attaining quasi-3D accuracy by means of refined structural models for *multi-field problems*, with specific developments for weakly coupled thermo-mechanics and strongly coupled piezoelectricity [B4⋆, A23⋆, A10, A9⋆]

2. The implementation of a global-local modelling approach upon coupling beam models with incompatible kinematics: this allows to limit the use of high-order models to regions where a refined quasi-3D solution is desired [PhD2, A16⋆]

3. The development of a Reduced Order Modelling based on the Proper Generalised Decomposition aiming at a reduction of the computational cost of detailed quasi-3D models. This development has been mainly carried out by my colleagues P. Vidal, O. Polit and L. Gallimard with only marginal contributions from my part; it is included in this document for the sake of completeness, because it represents an interesting approach that is built upon the core competences to which I could contribute.

Plate/shell FE with multi-field capabilities

Multi-field problems call for computational tools that are capable of grasping both, the local strain/stress state and the global structural response that arise from the local material deformation: different elastic and geometric coupling effects may arise that deserve an effective *structural* model for predicting the global deformed shape. This aspect has been put forward in our contribution to the Encyclopedia of Thermal Stresses [B4⋆] by means of results obtained with a reliable "geometrically exact" shell FE based on a refined Sinus model. The employed refined Sinus model retains the full 3D constitutive law by means of a quadratic assumption for $u_3(z)$ [A10], which yields a more accurate representation of the transverse deformation, see also the seminal work by Hildebrand *et al* [68], the contributions by Tessler [135, 136] and the discussion about Poisson locking by Carrera and Brischetto [31]. It should be noted that the above works assumed simplified temperature distributions, which are uniform over the in-plane domain and constant or at most linear across the thickness: temperature distributions obtained upon solving the heat equation over the heterogeneous cross-section may require an adequate refinement of the kinematics for coping with the more complex gradients [27].

Piezoelectric coupling introduces even more difficulties because the strongly coupled electrical and mechanical fields require matching approximations for enabling actuator and sensor applications, whose choice depends on the actuation mode. The use of patches of finite extension, the presence of geometrical coupling due to shell curvatures and possibly large deflections, are complicating aspects that challenge the formulation of reliable and effective shell FEM [13, 90, 133, 139].

A 7-parameter shell FE based on the $\{1{,}2\}$ theory of [68, 135] and with a LW quadratic approximation for the electric potential has been developed in the framework of the PhD thesis of M. Ben Thaïer [PhD1, A9⋆]. The electrostatic potential has been taken constant over the elementary domain in order to easily enforce the equipotential condition. Plate FEs based on a class of refined Sinus models including MZZF have been developed in [A23⋆] upon adopting a dedicated approximation for the electric field: the quadratic distribution of the electrostatic potential across the thickness of the piezo-layer is defined in terms of a Hermite interpolation rather of a Lagrange interpolation, i.e., in terms of the normal components at the electrodes and the physically relevant potential difference [73]; this latter variable has been again taken as constant over the elementary domain.

The XVF approach for coupling incompatible kinematics

Local 3D stress states occur usually over relatively small portions of the composite structure due to local load introductions or the presence of stress risers such as geometric and/or material discontinuities. Therefore, the effectiveness of FEM can be substantially enhanced by adopting "simple" structural models sufficient for grasping the global response over large domains, and by limiting computationally expensive quasi-3D models to the small regions in which an accurate resolution of the local response is demanded.

Among the several approaches available in open literature, our attention shall be focused on those allowing a monolithic and direct solution of all subregions – i.e., without requiring successive computations nor iterative schemes – and those coupling iso-dimensional elements that differ in the kinematics used to describe the model – i.e., transition elements coupling structural (1D or 2D) elements with solid (3D) elements fall out of the present scope.

Such a "simultaneous multiple model" approach has been devised by Robbins and Reddy [121] for coupling variable kinematics displacement-based plate elements in conjunction with a mesh superposition scheme; the generalised displacements of the refined model that are omitted in the coarse model are simply set to zero. The Arlequin method [11] has been employed by Biscani *et al.* for coupling CUF-based beam and plate elements [16, 17]: the Lagrange multiplier field has been expressed in CUF and the "fundamental nucleus" of the coupling operator has been derived. It is worth noticing that in [17] the method has been extended to mixed RMVT-based models as well. These approaches require a certain domain overlap and allow to deal with incompatible meshes. Strategies based on transition elements allow to couple subdomains without the additional computational cost of overlapping meshes. In their development of *hybrid* RMVT-based elements for an effective stress analysis in composite structures, Feng and Hoa formulated a transition element for coupling a first-order ESL plate/shell element with a LW element [61]. Ahn and co-workers coupled PVD-based ESL and LW models for plates within a transition element based on the $p-$version of FEM [2, 3]. In the framework of CUF, Carrera and co-workers developed transition elements in which each node may have a different kinematics and the coupling is automatically given by the stiffness matrix of the transition element [42]. This so-called node-dependent kinematics technique has been applied to beam, plate and shell FE, including mixed RMVT-based elements, and implemented within a $p-$version of FEM in order to locally refine the in-plane gradients also, see, e.g., [82, 95, 156].

Within the PhD thesis of C. Wenzel [PhD2] we adopted the general eXtended Variational Formulation (XVF) developed by Blanco *et al.* [18], which allows to link incompatible models directly at the interface between adjacent subdomains. The absence of mesh overlap and transition regions minimises the input required by the user for setting up the global-local model. The XVF introduces two fields of Lagrange multipliers for merging the incompatible kinematics in weak sense through integration over the interface: simple 2D (for beams) and 1D (for plates/shells) integrals need thus to be evaluated. The two Lagrange multiplier fields are defined in the spaces corresponding to the *simple* and *refined* kinematics to be coupled. A scalar coefficient $\gamma \in [0, 1]$ allows to distribute the weight of each field within the coupling operator, whose contribution to the displacement-based variational statement can be written over the coupling interface Γ_c as

$$\delta\Pi^{\text{coupling}}(u_i^s, u_i^r, \lambda_i^s, \lambda_i^r;\, \delta u_i^s, \delta u_i^r, \delta\lambda_i^s, \delta\lambda_i^r) = \\ \gamma \int_{\Gamma_c} \lambda_i^s(\delta u_i^s - \delta u_i^r)\,\mathrm{d}\Gamma \;+\; (1-\gamma)\int_{\Gamma_c} \lambda_i^c(\delta u_i^s - \delta u_i^r)\,\mathrm{d}\Gamma \\ +\gamma \int_{\Gamma_c} \delta\lambda_i^s(u_i^s - u_i^r)\,\mathrm{d}\Gamma \;+\; (1-\gamma)\int_{\Gamma_c} \delta\lambda_i^c(u_i^s - u_i^r)\,\mathrm{d}\Gamma \quad (4.4)$$

The projection between the 2 spaces u_i^s and u_i^r, which correspond to those of λ_i^s and λ_i^r, is defined by splitting the *refined* kinematics into a "parallel" and an "orthogonal" contribution with respect to the *simple* kinematics, $u_i^r = u_{i\|}^r + u_{i\perp}^r$, and upon imposing the orthogonality condition

$$\int_{\Gamma_c} \delta\lambda_i^s\, u_{i\perp}^r\,\mathrm{d}\Gamma = \int_{\Gamma_c} \delta\lambda_i^s\,(u_i^r - u_{i\|}^r)\,\mathrm{d}\Gamma = 0 \quad (4.5)$$

The prototypical implementation of this approach has been carried out by C. Wenzel in EvalEF for beam elements and first results have been published in [A16⋆] : the flexibility of the method has been demonstrated by coupling Euler-Bernoulli and Timoshenko beam elements with refined Sinus-based shear deformation beam elements with and without the transverse normal deformation. It turned out that, as already pointed out by Hu *et al.* in the framework of Arlequin method [69], when dealing with ESL models of highly heterogeneous structures such as sandwich sections, the coupling operator should be enhanced upon including a weighting coefficient corresponding to the stiffness of the plies within the integrals of Eq. (4.4). Since the results are very promising, an extension of the XVF approach to SGUF plate models is being considered within the in-house code developed by G. Di Cara.

Reduced order modelling by means of the PGD

In view of reducing the computational cost associated to high-fidelity analyses of composite structures, a Reduced Order Modelling approach has been developed based on the Proper Generalised Decomposition (PGD) method [98] and relying on the experience gathered in the field of refined beam/plate/shell models and of reliable FE procedures. The main idea, which can be also found in the early work by Savoia and Reddy [127], is to invoke the variables separation for decomposing the solution as a product of gradients along the in-plane coordinates $\hat{\mathcal{U}}^i(x_1, x_2)$ and those across the composite stack $f_{\mathcal{U}}^i(z)$:

$$\mathcal{U}(x_1, x_2, z) = \sum_{i=1}^{N} f_{\mathcal{U}}^i(z)\,\hat{\mathcal{U}}^i(x_1, x_2) \quad \text{with} \quad \delta\mathcal{U} = \sum_{i=1}^{N} \left(\delta f_{\mathcal{U}}^i\right)\hat{\mathcal{U}}^i \;+\; f_{\mathcal{U}}^i\left(\delta\hat{\mathcal{U}}^i\right) \quad (4.6)$$

where the through-thickness distribution $f_{\mathcal{U}}^i(z)$ is unique for the whole domain (x_1, x_2) and, conversely, $\hat{\mathcal{U}}^i(x_1, x_2)$ is a scalar value distributed over the 2D domain (x_1, x_2). Each value i corresponds thus to a "couple" of solutions which are found individually based on the residuum defined by the $i-1$ terms. The resulting solution is herewith obtained by solving iteratively two separate problems of reduced dimensionality, instead of by a direct solution of the whole structural problem. A fixed-point algorithm is used for the iterative solution of each "couple". The in-plane solution is obtained by means of robust beam, plate or shell FEM, see, e.g., [144, 145], while the gradients across the thickness are defined by referring to high-order PVD or RMVT models, which are known to provide quasi-3D solutions according to our previous experience [A22, A17, 149, A31]. Despite the computational overhead introduced by the incremental-iterative scheme required to solve the 2 problems, the solution based on the variables separation can be more efficient when compared to solutions employing refined LW models over the whole FE mesh. The application to the strongly localised gradients of free-edge effects is of particular interest [142]. Furthermore, based on our previous developments and findings about multi-field problems, the approach has been successfully extended to deal with thermal loading [143] and piezoelectric coupling [72].

The solution "couples" define a reduced basis for parametric analyses [44], and the method has been further developed towards multi-parametric problems: additional variables are introduced upon which the solution will depend, such as design parameters (e.g., the orientation angles of each ply) or some probabilistic parameters (e.g., some stiffness and/or strength values) [64, 141]. The obtained parametrised solution constitutes a numerical design chart which can be subsequently used within, e.g., optimisation processes or reliability analyses: since the first step can be carried out *off-line*, the application-driven optimisation or reliability study can be performed at a very reduced computational cost. It may be finally interesting to note that the "couples" provide a hierarchy of gradients that contribute to the solution in a certain energy sense, hence identifying a sequential refinement that can be compared to the Axiomatic/Asymptotic Method [41] or the Variational Asymptotic Method [15].

I.5

Selected results

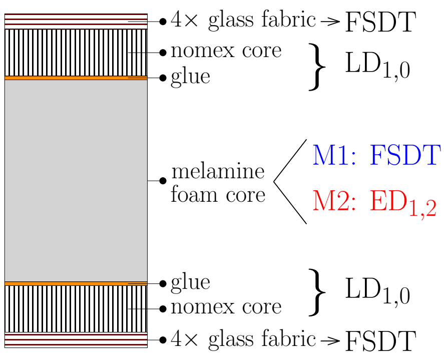
This Chapter presents some results for illustrating the developments presented so far. All selected case studies are available in the produced papers and PhD theses, only some results have been re-computed for the sake of clarity. Therefore, the presentation shall be limited to those details that allow appreciating the relevance of the considered examples.

I.5.1 Vibration and buckling of sandwich panels

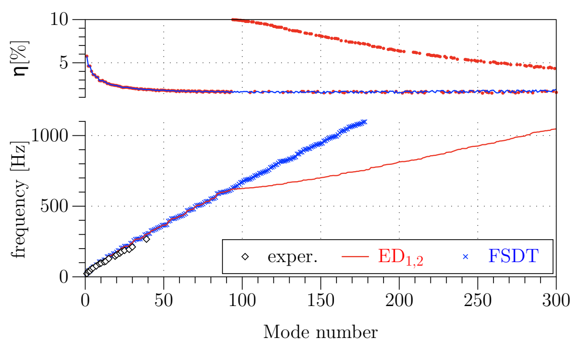
The transverse deformability localised within the relatively soft and thick core layers is the main driver for several characteristic behaviours that render sandwich panels particularly difficult to analyse. Transverse normal effects can play an important role even in the global response, as it will be highlighted by the thermal buckling response and by the free-vibration response of double-core sandwich panels. Moreover, the relatively soft core may promote a short wavelength response whose characteristic length is of the order of the thickness of the panel: some examples are shown of the studies carried out concerning this so-called wrinkling instability.

Vibration of a double-core sandwich plate

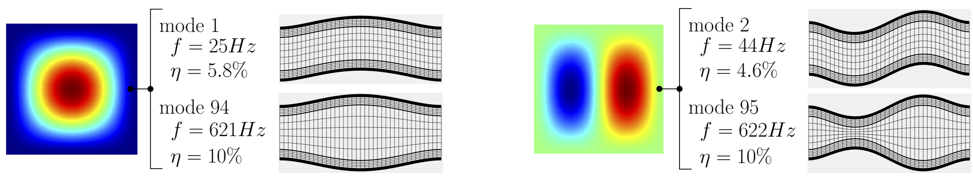
Due to the high stiffness and low mass density, composite structures show often unsatisfactory vibration performances. The Constrained Layer Damping (CLD) relies on the deformation of a soft, viscoelastic layer within the stack and is the main mechanisms for enhancing the damping properties. For conventional CLD it is sufficient to take into due account the transverse shear deformability of the viscoelastic layer. However, double-core sandwich structures can achieve even superior damping effects upon activating thickness-stretch modes and double-wall resonances. So, the "Helicopter Garteur Action Group 20" suggested the innovative trim panel illustrated in Figure I.5.1a for reducing cabin noise. This configuration has been studied in the paper [A33⋆] by using Ritz expansion orders $R = S = 26$ and 2 SGUF models: both regroup the 13 plies into 5 sublaminates, the only difference is in the kinematics adopted for the layer consisting of the thick melamine foam core: model M1 uses FSDT, while $ED_{1,2}$ is used in M2. The modal frequencies and associated loss factors for the first 300 modes are reported in Figure I.5.1b, where the experimental data reported in [130] are included for comparison. The frequency spectrum in Figure I.5.1b clearly shows a discrepancy between the two models for $f > 620$ Hz, starting from which M2 shows a higher modal density compared to M1. In fact, long wavelength *symmetric thickness-stretch* modes driven by the melamine foam appear for $f > 620$ Hz, which enrich the spectrum of short wavelength *antisymmetric thickness-shear* modes obtained with the classical model M1, as exemplarily illustrated in Figure I.5.1c. Due to the deformation of the melamine foam core these thickness-stretch modes are characterised by higher loss factor, which is captured only by the model M2, see Figure I.5.1b (top). Therefore, the use of shear-deformation theories neglecting the transverse normal deformation may not be sufficient to properly investigate the acoustic properties of viscoelastic structures because thickness-stretch modes may occur in the frequency band of interest.



(a) Double-core sandwich and SGUF models



(b) Modal eigenfrequencies and loss factors

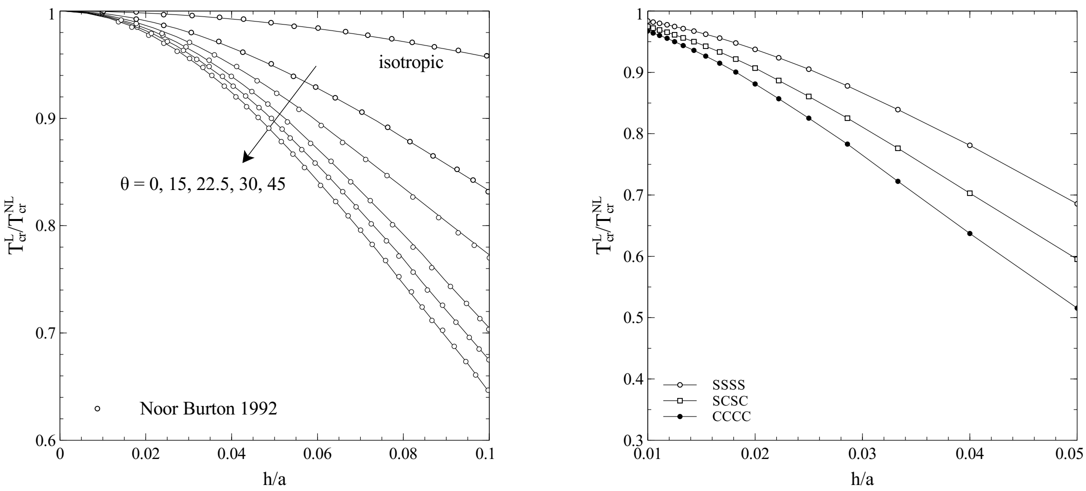


(c) First 2 modes sharing the in-plane pattern: thickness-stretch mode occurs at $f > 620$ Hz

Figure I.5.1 – Free-vibration analysis of an innovative trim panel – (a) double-core sandwich configuration; (b) eigenfrequencies and loss factors for the fully clamped square plate (a = 840 mm, h = 21.68 mm); (c) modal shapes with same in-plane pattern but different thickness modes.

Thermal buckling

As already pointed out, thermal loading produces effects in the transverse direction that are of the same order of magnitude than those in the in-plane directions. Therefore, the effect of thermal expansion of transversely soft and thick plies, such as core layers in sandwich panels, deserves a particular care. In the paper devoted to the thermal buckling of composite structures [A26], the contribution of the transverse deformation to the thermally induced pre-buckling load has been emphasised: Figure I.5.2 displays exemplarily this effect, where the ratio T_{cr}^{L}/T_{cr}^{NL} is reported with varying h/a ratios for a square laminate and a square sandwich. T_{cr}^{NL} denotes the buckling temperature obtained upon including the transverse normal deformation in the definition of the initial stress[1], while T_{cr}^{L} is the buckling temperature obtained by retaining only the in-plane thermal expansion. The results show that T_{cr}^{L} always overestimates the critical temperature. The influence of the transverse deformation obviously increases with the thickness of the plate and it is much larger for the sandwich than for the laminated plate.



(a) Laminate $[\pm\theta]_5$, SSSS

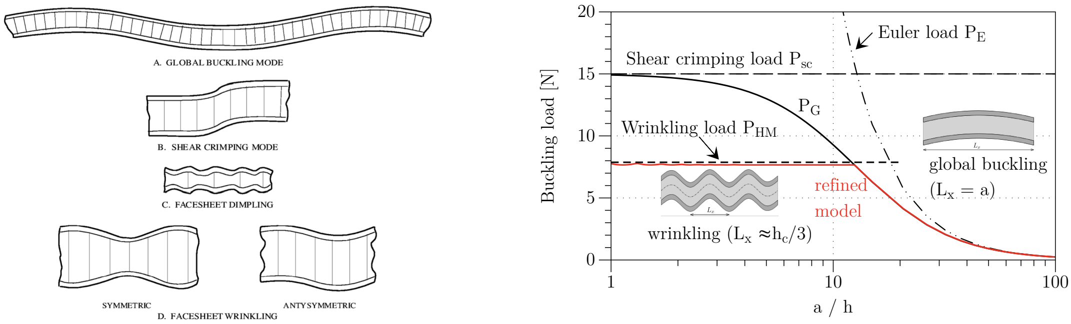
(b) Sandwich $[(\pm 45)_5/c/(\mp 45)_5], h_f/h = 0.1$

Figure I.5.2 – Influence of the transverse deformability for the thermal buckling problem: anisotropic laminate (left) and sandwich plate (right). Results taken from [A26].

Buckling and wrinkling of sandwich plates

In sandwich panels, the elastic instability can occur at different length scales, as illustrated in Figure I.5.3a, depending on the geometric and material properties of core and faces. Dedicated analysis tools or design formulas are usually employed for the global buckling ("structural" failure) and short wavelength wrinkling (often assimilated as "material" failure [49]). A variable kinematics approach in conjunction with an efficient solution method for grasping the short wavelength response can be, hence, advantageously used for all configurations. This is exemplarily shown in Figure I.5.3b, where a refined model is shown to detect the transition from global buckling to wrinkling and to well agree with dedicated formulas.

[1]The pre-buckling state is defined as ${}_0u_\alpha(x_i) = 0, {}_0u_3(x_i) = {}_0u_3(z)$, with ${}_0u_{3,3}$ obtained from the definition of the non-linear Green-Lagrange strains ${}_0\epsilon_{33} = {}_0u_{3,3} + \frac{u_{3,3}^2}{2}$ [97]



(a) Buckling modes of sandwich panels [80]

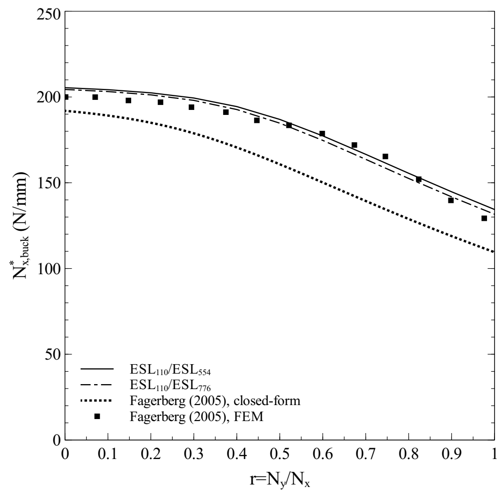
(b) Buckling loads and modes depending on a/h

Figure I.5.3 – Long and short wavelength instability of sandwich panels (left). Wrinkling is the dominant mode for short struts with thin faces: while different design formulas are used for different modes, a unique refined model is capable of grasping the dominating instabilities throughout all slenderness ratios (right). The employed formulas are [5]: $P_E{=}\pi^2 EI/a^2$, $P_{sc}{=}GA$, $P_G{=}P_E/(1{+}P_E/P_{sc})$,$P_{HM}{=}0.91\,A\,\sqrt[3]{E_x^f\,E_z^c\,G_{xz}^c}$.

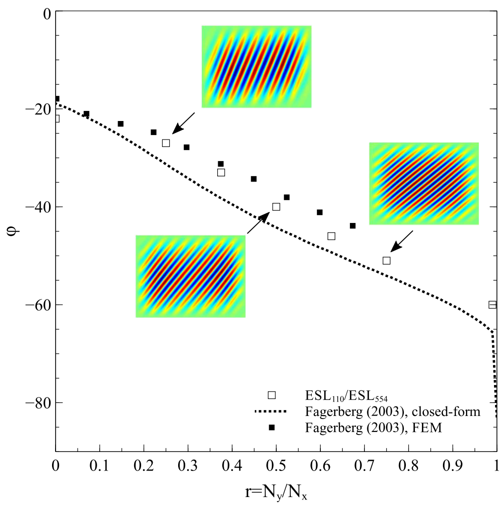
Several studies about wrinkling could be successfully carried out by referring to Navier solution because they involved only orthotropic materials and the periodic short wavelength response resulted barely dependent on the boundary conditions [A13⋆] : these studies allowed to recover the reference solutions, usually obtained with 3D FEM [A21⋆] , and to provide insight into the range of validity of relatively simple analytical formulas that are currently employed for the sizing of sandwich panels [A34]. For more challenging configurations, for which Navier solution is no longer suitable, the Ritz method could be successfully employed. The key for this is the excellent stability of the implemented solution method, which allows using high polynomial orders to grasp the short wavelength wrinkling patterns. An example extracted from [A28⋆] is reported in Figure I.5.4, where the elastic stability under biaxial compression is studied for a thick sandwich ($a = 200, b = 150, h = 50$ [mm]) with thin composite faces ($h_f = 0.5$ mm) whose fibres are oriented at $\theta = 15°$. The configuration was investigated by Fagerberg [60], who provided a closed-form approximate solution and 3D FE results. An excellent agreement with the 3D FEM is found with Ritz expansion orders $R = S = 50$ and an SGUF model adopting FSDT ($ED_{1,0}$) for the faces and very high-order models for the core: note that ED_{554} or ED_{776} are required for capturing the complex deformation of the anisotropic wrinkling pattern.

I.5.2 Piezoelectric coupling

For an accurate design of smart structures with piezo-layers embedded within the load path, it is necessary to evaluate both, the global structural response and the local gradients arising in consequence of the applied load, the electrical boundary conditions and the piezoelectric coupling.



(a) Axial buckling load



(b) Skew angle φ of the wrinkling waves

Figure I.5.4 – Wrinkling analysis for a thick soft-core sandwich with anisotropic faces under biaxial compression defined by the ratio $r = N_y/N_x$. Results extracted from [A28⋆] .

A first example of the strong coupling that characterises piezoelectric interactions is illustrated in Figure I.5.5, which reports the first 6 bending modes in the $(x_1, z)-$plane of a thick cantilever plate (a/h = 4) constituted of an elastic, orthotropic bottom ply (GFRP) and a piezoelectric (PZT-5A) top ply (data provided in [140]). The piezoelectric ply is covered at its four edges by electrodes in short-circuit ($\Phi = 0$). The reported results are obtained by the Ritz method [IC24] and one notices again the perfect agreement with the reference 3D FEM. The modal plots of the electrostatic potential illustrate clearly how the electrical response is coupled with the mechanical vibration mode: high-order models are required for resolving the local gradients of the electrical and mechanical response.

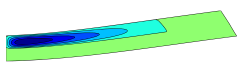
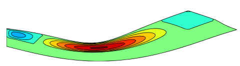
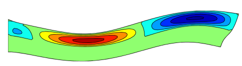
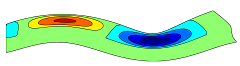
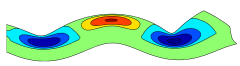
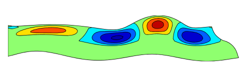
Mode 1	Mode 2	Mode 3
1.1890 (1.1890)	4.8364 (4.8363)	8.5873 (8.5871)
		
Mode 4	Mode 5	Mode 6
10.618 (10.618)	16.360 (16.359)	19.852 (19.851)
		

Figure I.5.5 – Modal analysis of a thick cantilever GFRP/PZT-5A plate in cylindrical bending: non-dimensional eigenfrequencies $\omega \frac{a^2}{h} \sqrt{(\rho/C_{11})_{PZT}}$ and corresponding deformed shape with plots of the electrostatic potential $\Phi(x_1, z)$. Results obtained with LD7 and Ritz method $R = 20, S = 1$, values in parentheses obtained by Abaqus with 160×40 CPE8E elements.

A second example, taken from [A9⋆], is shown for highlighting how strong the location of the piezoelectrically induced deformation influences the resulting structural response. A simply-supported laminated $[0/90]_s$ cylindrical panel hosts a PZT-4 layer, whose position is either at the bottom of the stack ($[p/0/90/90/0]$), or at its center ($[0/90/p/90/0]$) or at its top ($[0/90/90/0/p]$), see Figure I.5.6 (left). The piezo-layer is actuated in 31-mode ($e_{31} = -5.2\,\text{C/m}^2$) by a uniform electric field $E_z = -400\,\text{V/mm}$: the resulting distribution of the non-dimensional radial displacement $u_3(\theta)/h$ for the 3 configurations is illustrated on the right of Figure I.5.6. Good agreement of our developed shell FE is found with the shell FE of Saravanos (1997) and Balamurugan & Narayanan (2008) as well as with 3D FEM computations. It is obvious that due to the interplay of shell curvature and anisotropy, very different deformed shapes are obtained depending on the position of the actuator, hence the interest in employing reliable structural FE with piezoelectric capabilities.

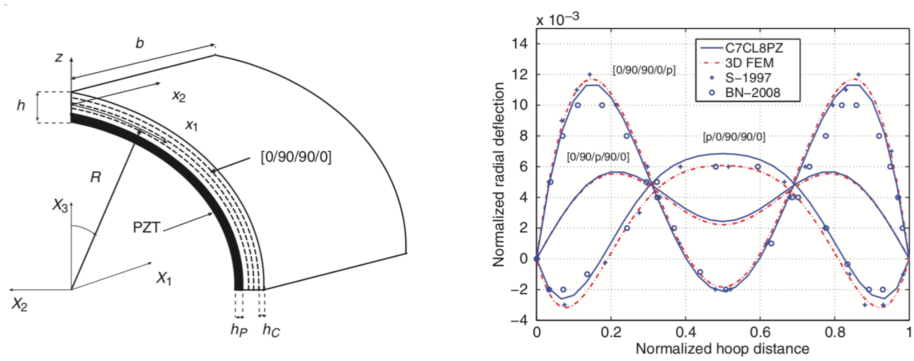
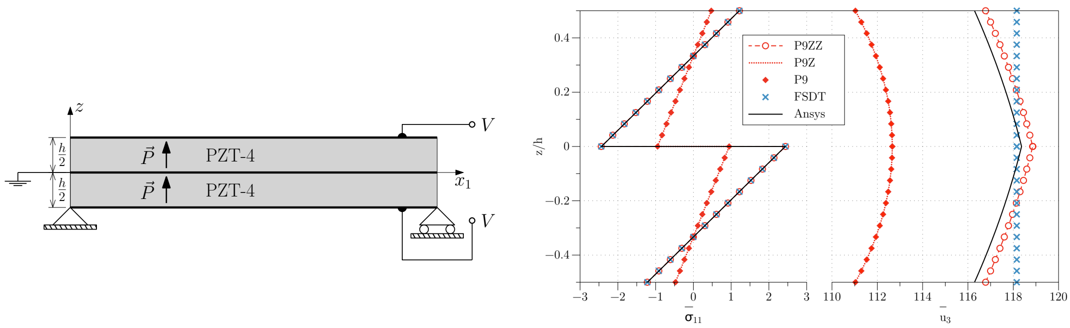


Figure I.5.6 – Composite cylindrical panel with piezoelectric actuator: the configuration $[p/0/90/90/0]$ is depicted on the left; the radial deflection along the hoop direction is displayed on the right for three different positions of the piezoelectric actuator. Figures extracted from [A9⋆] .

A final result is taken from [A23⋆] in order to emphasise a non-classical local effect that may arise due to the piezoelectric coupling. The parallel bimorph configuration illustrated in Figure I.5.7a is analysed by means of three refined plate models within a FEM approach: a refined 9-parameter Sinus model with a quadratic transverse displacement for including the 3D constitutive law [147] (referred to as P9 model), to which MZZF is superimposed either only to the in-plane displacements u_α (P9Z model), or to *all* displacements, i.e., the transverse displacement u_3 as well (P9ZZ model). In the actuator configuration, the same voltage $V = 50\,\text{V}$ is applied to the outer electrodes while the mid-electrode separating the two PZT plies is grounded ($V = 0$), thus producing an electric field of equal intensity but opposite direction in the top and bottom plies. Since the top and bottom plies have parallel polarisation, schematically represented by $\vec{P}$ in Figure I.5.7a, a bending motion occurs.

The resulting bending stresses obtained by the models is reported in Figure I.5.7b, where one remarks that the P9 and P9Z produce identical and erroneous results compared to the FSDT, P9ZZ and 3D FEM results. The reason is to be found in the opposite transverse normal deformation ϵ_{33} induced in the two piezo-layers by the opposite electric field: if the 3D piezoelectric constitutive law is retained, it is *mandatory* to allow a slope discontinuity across the thickness for the transverse normal displacement. This is accomplished by a LW description of the kinematics (3D FEM) or adding the MZZF to the transverse displacement $u_3(z)$ as it is done for the P9ZZ model, as it is clearly visible in Figure I.5.7b. FSDT does not "see" this effect because it uses the reduced 2D plane-stress constitutive law.



(a) Parallel bimorph actuator

(b) Bending stress and deflection

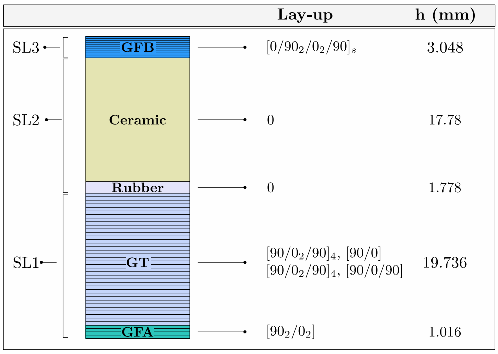
Figure I.5.7 – Piezoelectric bimorph in actuator configuration: mechanical response for $\frac{a}{h} = 10$ [A23⋆] .

I.5.3 Local stress analysis

Some examples are finally presented that concern the accuracy of the local stress response in composite structures. A first example considers the bending of a complex composite stack and allows to emphasise the interest of the SGUF approach for reducing the model size without affecting the accuracy. The attention is subsequently turned towards some examples of very localised free-edge effects. Since the interest is focused on an accurate 3D stress analysis, the results of RMVT-based models will be emphasised because they allow to assure a physically meaningful interlaminar continuous transverse stress field. The high computational cost of the free-edge analyses and the localisation of the 3D stress state into small regions motivates the final application of the global-local modelling approach implemented within the XVF framework.

Bending of the CAV panel

A case study dealing with a complex composite panel inspired from those of the US Army "Composite Armour Vehicle" has been proposed by Cho & Averill [45], for which they also provided an exact elasticity solution. It consists in a simply-supported square plate of size $a = 172\,\text{mm}$ whose composite stack is illustrated in Figure I.5.8a, subjected to a bisinusoidal pressure load. This case study has been thoroughly investigated in [A20⋆] . The proposed subdivision into three sublaminates is reported in Figure I.5.8a and follows the functional structure of the CAV panel: SL1 for the inner shell, SL2 for the armour, and SL3 for the outer composite shell.



(a) Stack of the CAV plate [A20⋆]

	SL1	SL2	SL3	DOF
M1	$\mathrm{EM}^{2,1}_{1,0}$	$\mathrm{M}^{\mathrm{L2,E1}}_{\mathrm{L1,L1}}$	$\mathrm{EM}^{2,0}_{1,0}$	29
M2	$\mathrm{EM}^{2,3}_{1,0}$	"	"	31
M3	"	"	$\mathrm{EM}^{2,2}_{1,0}$	33
LM2	LM2	LM2	LM2	661

(b) RMVT models used in each sublaminate

Figure I.5.8 – CAV panel: definition of the sublaminates and of the mixed SGUF models.

While the results in [A20⋆] were obtained by displacement-based models, the results discussed in the following refer to RMVT-based mixed models: the CUF model LM2 and three SGUF models with increasing number of DOF have been considered, labeled M1 to M3, and defined in Figure I.5.8b. It is worth recalling that in EM-models, the ESL description applies to the transverse stress variables as well. An LW description is adopted for the displacement and transverse shear stress variables only for the two-ply armour (SL2) in order to account for the strong mismatch between the rubber and the ceramic ply. Furthermore, the homogeneous stress boundary conditions at the top and bottom surfaces have been exactly enforced, i.e., $\sigma_{\alpha 3}(z = \pm\frac{h}{2}) = \sigma_{33}(z = -\frac{h}{2} = 0)$, whereas the applied pressure intensity at the top has *not* been enforced to the field variable σ_{33}.

The solution has been computed with Ritz expansion orders $R_U = S_U = 10$ and $R_S = S_S = R_U+1$. The local response is reported in Figure I.5.9 in terms of the in-plane bending stress and the transverse shear and normal stresses: these latter variables are directly obtained from the solution vector of the mixed models. An overall good agreement with the reference solution of Cho & Averill is easily recognised. In particular, the ESL description for the transverse stresses of M1 and M2 are smooth and provide an excellent approximation of the reference solution. By virtue of the cubic assumption, the model M2 improves M1 with respect to $\sigma_{33}(z)$ in the bottom sublaminate SL1. On the other hand, the quadratic assumption for σ_{33} introduced in SL3 for the model M3 is seen to degrade the accuracy because a spurious oscillation occurs due to an excess of stress variables. Undesired oscillations of smaller amplitude are also visible in the CUF model LM2, even in the bending stress.

From this example it is concluded that RMVT models *can* be effectively used to improve the transverse stress field, provided the appropriate assumptions are used for the stress variables

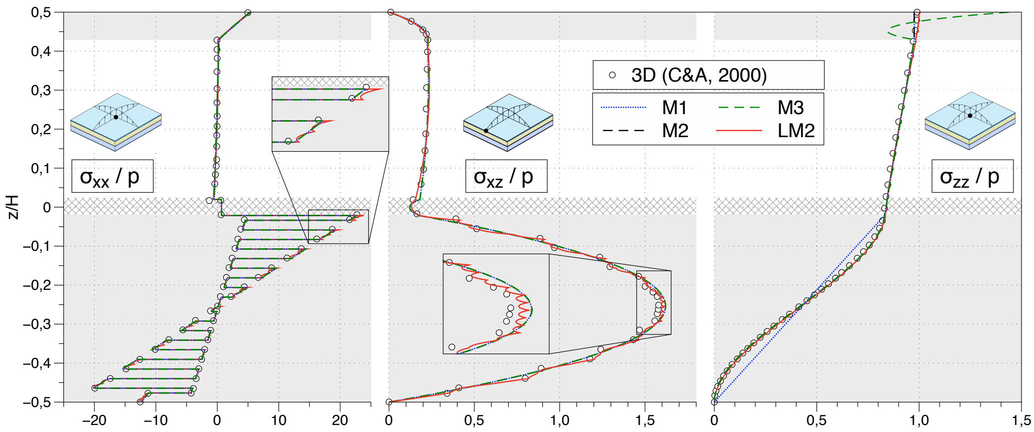


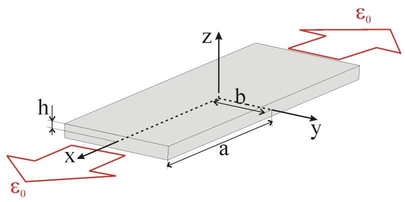
Figure I.5.9 – Local response of the CAV panel: reference solution of Cho & Averill [45] and present Ritz solution for the 4 different RMVT models defined in Figure I.5.8b.

with respect to the chosen kinematics. Results obtained by PVD-based models adopting the same kinematics can reach an accuracy similar to that of the models M1 and M2 only if the transverse stress field is computed from the equilibrium equations in a post-processing step, as it has been done in our paper [A20⋆] . This post-processing procedure is obviously quite straightforward in the framework of Ritz method, where the in-plane gradients are resolved by high-order polynomials, but it can be a more difficult task if low-order FE are used.

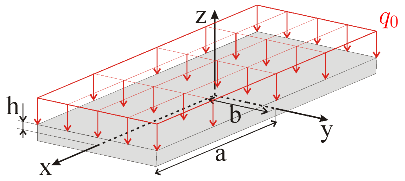
Free-edge effects

CUF-based plate FE have been applied to the free-edge problem for assessing their capability to accurately resolve the steep gradients that characterise the local 3D stress response with simple 2D meshes. The considered problems are depicted in Figure I.5.10; the analyses have been carried out with the Abaqus User Element implementation in the framework of the PhD theses of C. Wenzel [PhD2] and T.H.C. Le [PhD4]. High-fidelity 3D FEM solutions have been also computed, where the same in-plane mesh has been used (see Figure I.5.10c) and each ply has been discretised with 8 or 12 quadratic solid elements across its thickness. Symmetry conditions have been exploited whenever possible for reducing the computational domain.

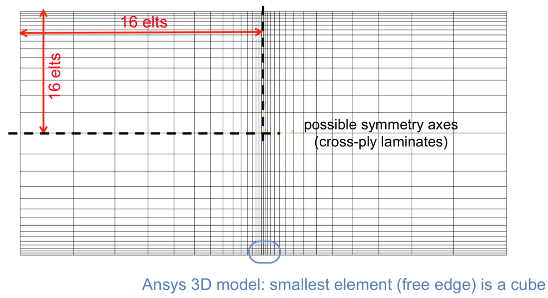
The paper [A11⋆] deals with the well-known Pipes-Pagano problem of *symmetric* laminates subjected to a uniform tension load, see Figure I.5.10a. The refined Sinus model of [A10] and all CUF-models based on PVD and RMVT have been employed to compute the transverse stress distributions. Results obtained with LM4 are exemplarily reported in Figure I.5.11 for the 4-layer cross-ply laminate and show a good accuracy with respect to several reference solutions. It should be noticed that the through-thickness distributions of the LM4 model are interlaminar continuous but display some spurious oscillations when compared to the 3D FEM solution. Results obtained for other lamination schemes (the angle-ply $[\pm 45]_s$ and the quasi-isotropic laminates $[90/0/\pm 45]_s$, $[\pm 45/0/90]_s$) can be found in [A11⋆] .



(a) Free-edge – uniform tension [106]

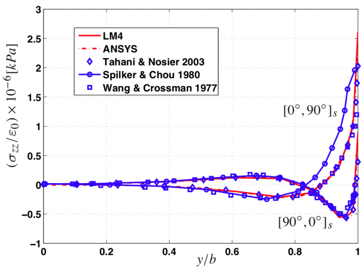


(b) Free-edge – bending load [134]

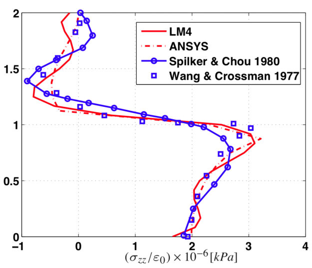


(c) FE mesh for the computation of free-edge effects

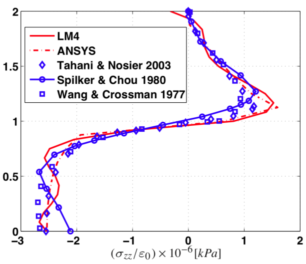
Figure I.5.10 – Free-edge problems: (a) laminate in uniform uniaxial tension; (b) laminate bent by a uniform pressure load (edges $x = \pm a$ are simply-supported); (c) in-plane FE mesh used for both configurations.



(a) $\sigma_{33}(y, z = 0)$



(b) $[0/90]_s$: $\sigma_{33}(y = 0.999b, z)$



(c) $[90/0]_s$: $\sigma_{33}(y = 0.999b, z)$

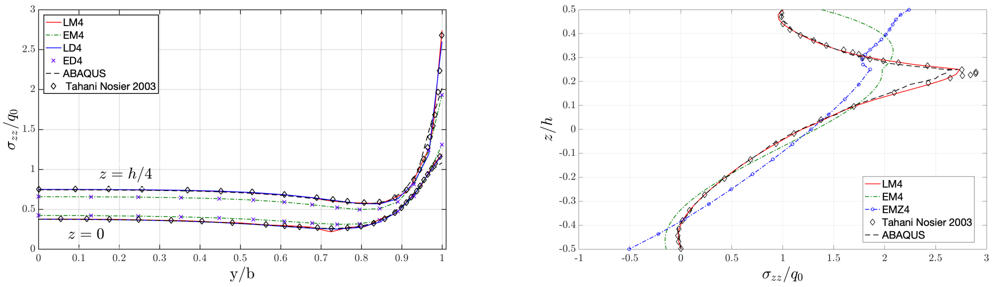
Figure I.5.11 – Free-edge effect for the Pipes-Pagano cross-ply configurations [A11⋆].

The free-edge effects arising under tension load have been compared to those occurring under bending load in the framework of the PhD thesis of C. Wenzel [PhD2], see also [A18⋆]. For the configuration depicted in Figure I.5.10b, the same mesh of Figure I.5.10c is used but the symmetry of the laminate about its mid-plane $z = 0$ can no longer be exploited for reducing the computational effort. The free-edge effect arising in the asymmetric $[90_3/0]$ cross-ply laminate subjected to a uniform pressure q_0 is displayed in Figure I.5.12: the bi-material interface at $z = h/4$ shows obviously the steepest gradient towards the free edge.

Furthermore, a post-processing step has been implemented for representing the stress singularity that characterises the free-edge effect: within a normalised distance $r \in]0, 1]$ from the free edge, a singular stress component σ_{ij} is represented as

$$\sigma_{ij}(r) \approx A_{ij}\, r^{-\alpha_{ij}} \quad \Leftrightarrow \quad \log \sigma_{ij}(r) \approx \log A_{ij} - \alpha_{ij}\, \log r \tag{5.1}$$

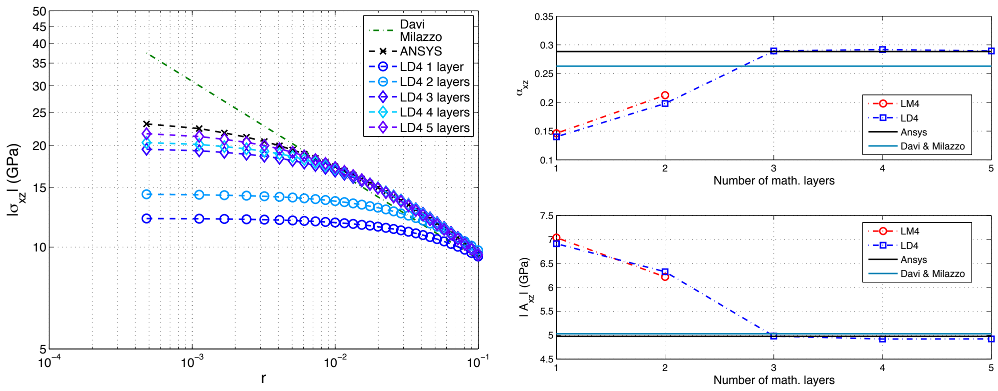
from which the estimates of the singularity power α_{ij} and its strength A_{ij} can be extracted



(a) $\sigma_{33}(y, z = 0)$ for $[0/90]_s$ and $[90/0]_s$ (b) $[0/90]_s$: $\sigma_{33}(y = 0.999b, z)$, $z \in [0, \frac{h}{2}]$

Figure I.5.12 – Free-edge effect for a $[90_3/0]$ laminate in bending.

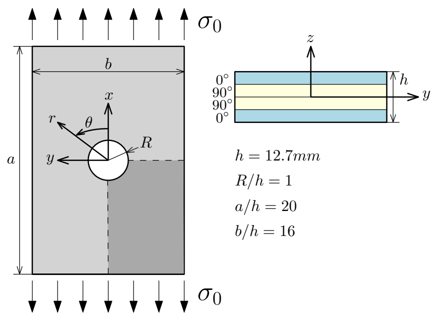
[A18⋆] . The solutions obtained by Davì & Milazzo with the BEM approach [50] have been taken as reference. Figure I.5.13 displays results for the $[\pm 45]_s$ Pipes-Pagano configuration: the convergence study of the singularity parameters α_{13} and $|A_{13}|$ with respect to the number of mathematical layers used in each physical ply shows that at least 3 numerical layers are necessary for correctly capturing the singularity.



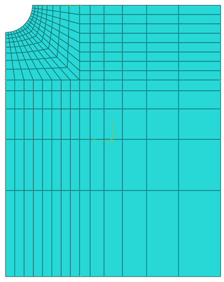
(a) $\sigma_{13}(r, z = \frac{h}{4})$ in log-log scale (b) Convergence of singularity parameters

Figure I.5.13 – Free-edge singularity for the Pipes-Pagano $[\pm 45]_s$ configuration.

Based on these promising results, T.H.C. Le applied the 8-node CUF-based plate FE to the analysis of the stress concentration induced by a central hole in a laminated plate subjected to uniaxial tension [PhD4]. This problem is a classical benchmark for the virtual testing of composite structures [67]: since the outcome of any damage or failure criterion is inherently dependent on the quality of the computed stress (or strain) state, it is interesting to assess the accuracy of the local response predicted by high-order 2D elements for this configuration. Figure I.5.14 illustrates the investigated configuration proposed by Lucking *et al.* [86], along with the definition of the structured meshes used for the computations: the coarsest mesh A is illustrated in Figure I.5.14b, the refined meshes B, C . . . listed in Figure I.5.14c are constructed with the same partition of mesh A. Only the quadratic 8-node FE is used for assuring an accurate discretisation of the curved edge of the hole.



(a) Description of the configuration



(b) Mesh A

Mesh	No. FE	No. Nodes
A	320	1041
B	832	2625
C	1344	4193
D	2624	8097
E	4672	14305

(c) 8-node FE meshes

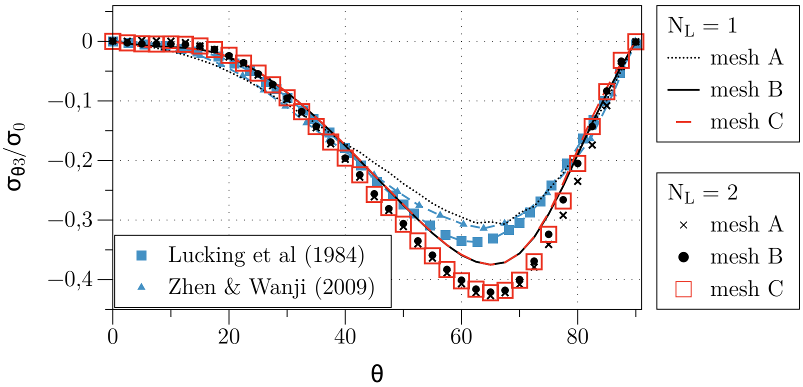
Figure I.5.14 – Cross-ply laminated plate with center hole in uniform tension and employed 8-node FE structured meshes.

The attention shall be restricted to the stress component experiencing the highest values, i.e., the interlaminar transverse shear strain $\sigma_{\theta 3}$ at the $z = h_p$ interface, where $h_p = \frac{h}{4}$ is the ply thickness. Figure I.5.15 and Figure I.5.16 show the distributions of $\sigma_{\theta 3}$ at a small radial distance $r^\star = R + \frac{h_p}{20}$ from the hole. Comparison with solutions available in literature is also reported [83, 86, 152]. Figure I.5.15a displays the circumferential distributions $\bar{\sigma}_{\theta 3}(r^\star, \theta, z = h_p)$ obtained with the mixed LM4 model for different meshes and different numerical layers N_L into which each ply has been subdivided: it appears that the refinement across the thickness by means of numerical layers has an important effect, while there is no improvement between the meshes B and C. A comparison between the displacement-based LD4 and RMVT-based LM4 models is reported in Figure I.5.15b for mesh B and $N_L = 2$: note that, despite the introduction of 2 numerical layers, the LD4 model does not fulfil the interlaminar continuity of the $\sigma_{\theta 3}$ at the interface $z = h_p$, whereas a unique value is obtained by the LM4 model.

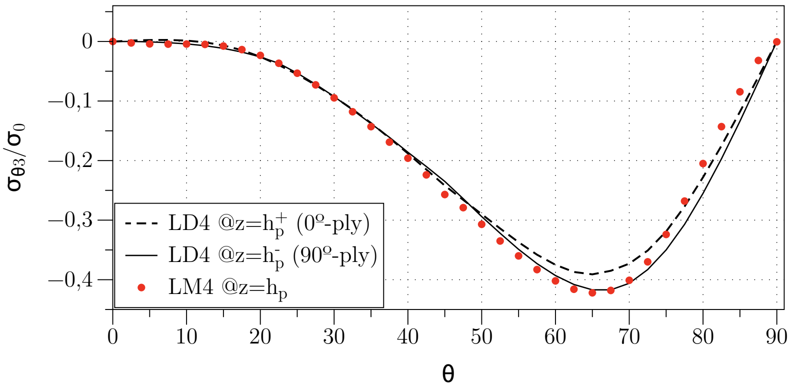
Through-thickness distributions of the transverse shear stress obtained by the LM4 model are reported in Figure I.5.16 for different angles θ, which show the steep gradient occurring at the interface $z = h_p$. The higher value of the interlaminar stress reached by using $N_L = 2$ numerical layers is clearly visible. However, some oscillations are visible in the curves in Figure I.5.16b with $N_L = 2$, whereas Figure I.5.16a with $N_L = 1$ show smoother distributions.

The above results all show that high-order plate models are capable of accurately capturing the rapid gradients of free-edge effects. However, despite the use of 2D meshes and of the Abaqus UEL implementation, important computational efforts are still required[2]. This motivates the further development of computational tools aiming at increasing the effectiveness of the models: the global-local approach based on the XVF [A16⋆] and the PGD approach [142] could introduce interesting savings in both RAM allocation and computational time.

[2]It should be noted that all computations have been performed on PCs and not on workstations

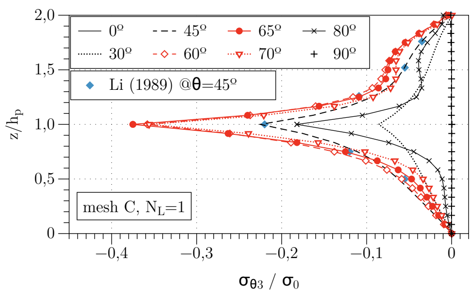


(a) Convergence study on $\bar{\sigma}_{\theta 3}(r^\star, \theta, z = h_p)$, LM4 model

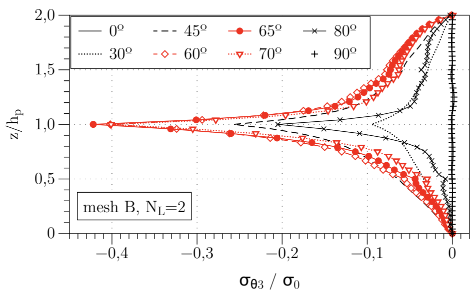


(b) Comparison of $\bar{\sigma}_{\theta 3}(r^\star, \theta, h_p)$ for LM4 and LD4 (mesh B, $N_L = 2$)

Figure I.5.15 – Laminate with hole: normalised interlaminar shear stress $\bar{\sigma}_{\theta 3}(\theta)$ at the interface $z = h_p$ and at the radial distance $r^\star$.



(a) mesh C, $N_L = 1$



(b) mesh B, $N_L = 2$

Figure I.5.16 – Laminate with hole: through-thickness distributions $\bar{\sigma}_{\theta 3}(z)$ at given angles θ and at the radial distance $r^\star$ (LM4 model).

Global-local modelling with XVF

The sub-modelling approach devised in the PhD thesis of C. Wenzel [PhD2] allows to couple FE with incompatible kinematics assumptions, thus allowing to limit the use of computationally expensive high-order models to those regions in which an accurate 3D stress response is requested, as in the free-edge region discussed above. The prototypical implementation of the XVF approach has been carried out for beam elements only and has not yet been applied to free-edge problems. Interesting and encouraging preliminary results have been nevertheless obtained for a simply-supported sandwich beam subjected to a localised pressure load at the center of its span [A16⋆] . Due to symmetry, the computational domain can be reduced to one-half of the beam, see Figure I.5.17, for which a regular mesh of 20 1D (beam) elements has been used. Two interfaces Γ_c are introduced to subdivide the domain into two "refined" regions Ω_r, where the loads are introduced and for which beam FEs with a refined Sinus kinematics (Sin-z2, [147]) are used, and one central "simple" region Ω_s, for which either Euler-Bernoulli or Timoshenko beam models are used.

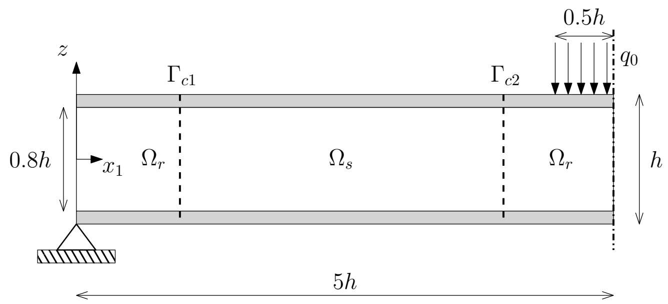


Figure I.5.17 – Problem description for the prototypical application of XVF: two interfaces Γ_c subdivide the domain into 3 regions with "simple" models in Ω_s and "refined" models in Ω_r.

Figure I.5.18 shows the normalised transverse shear stress $\bar{\sigma}_{13} = \sigma_{13}/q_0$ computed at $z = 0$ along the beam axis x_1. Only 2 Sin-z2 FE are used in each refined region Ω_r and 16 Timoshenko elements discretise the central region Ω_s. Two values for the coupling operator γ are considered and the attention is to be restricted to the region of interest Ω_r: it is seen that with $\gamma = 1$, only one element in Ω_r has a solution slightly worse than if the whole mesh is constituted of Sin-z2 elements. A different value for $\gamma \in [0, 1[$ is seen to pollute the solution over a certain distance in the regions of interest Ω_r. These results confirm that XVF is effective in reducing the computational effort in view of an accurate local stress response.

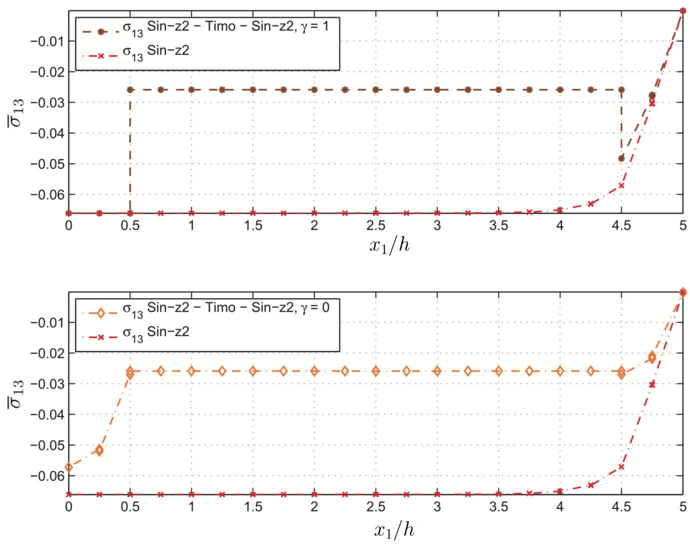


Figure I.5.18 – Transverse shear stress $\bar{\sigma}_{13}$ along the beam axis for XVF-coupled FE models: results with $\gamma = 1$ (top) and $\gamma = 0$ (bottom). Results extracted form [A16⋆] .

I.6

Conclusions and Outlook

Axiomatic structural models have been considered within the classical displacement-based variational framework as well as Reissner's mixed theorem RMVT that is expressly dedicated to composite stacks. The variable kinematics approach has been developed as a useful tool for enhancing the effectiveness of *structural* models of composites. On the basis of the seminal compact notation introduced by Professor Carrera, the SGUF is a *formal* extension of CUF allowing the use of independent models over arbitrarily defined numerical layers (the sublaminates) that compose the composite stack, thus limiting the use of refined models to selected regions. It is particularly suitable for dealing with complex stacks, such as sandwich-type constructions that are characterised by a strong mesoscale heterogeneity with alternating thick/thin and stiff/weak plies.

Besides the quasi-analytical Navier-type solution, useful for assessing some fundamental properties of the developed plate/shell models, a very efficient implementation of the Ritz method as well as a reliable FE discretisation have been considered for solving the 2D governing equations expressed in Unified Formulation.

Relevant applications have been presented that emphasise how the use of refined models and the full 3D constitutive law can be limited only to certain sub-regions of the stack, for instance the core of sandwich panels. Extensions to the (weak) thermo-mechanical and the (strong) piezoelectric coupling have been also considered, for which the local interactions are intimately related to the global structural response. The works dedicated to the local stress analysis have shown that it is possible to accurately recover high-fidelity 3D FEM solutions with 2D meshes and high-order plate elements, even for free-edge effects and in terms of stress singularity. The need for LW models is obviously confirmed and computational approaches aiming at reducing the cost associated to these models have been devised: a simultaneous global-local modelling approach based on XVF and the variable separation approach based on the PGD.

Further investigations are still necessary for a reliable and effective application of RMVT models to an accurate 3D stress response. The possibility of obtaining a unique value of the transverse stresses at bi-material interfaces directly from the solution of the FE computation is a major advantage of these models, in particular in view of the further use of the stress solution for delamination analysis. However, the superiority of mixed RMVT models over PVD is not as evident because mixed elements require a higher computational cost, in particular in terms of allocated memory, and because the resulting stress fields may show some spurious oscillations. The former issue could benefit from the mentioned model reduction approaches (XVF, PGD), which shall be further developed; the latter issue is related to the stability of mixed models and is currently being investigated [MT7, Rep1, Rep2]. The possibility offered by GUF and SGUF to introduce different descriptions for individual field variables is the key point for arriving at an "optimum" model. Furthermore, the availability of Navier, Ritz and FEM solution methods allows to investigate the role of various in-plane approximations on the stability of the resulting computational model.

The application of the proposed refined plate/shell models to the simulation of damage and failure is not expected to introduce additional complicating problems other than those that plague the conventional 3D FEM. Several regularisation approaches have been devised for contrasting damage/strain localisation and the consequent mesh-dependency of intralaminar (continuum) damage, such as non-local or gradient-enhanced models [103] or extended FE methods with the possibility of introducing discontinuities within the element [93, 94] ... For interlaminar damage and fracture, the Cohesive Zone Model [51, 138] or the combined stress-energy approach of the Finite Fracture Mechanics [91] appear most promising [B3].

The above developments are nevertheless useless without a robust and effective computational framework for non-linear problems. A first extension towards geometrical nonlinearity has been documented in this report, for which a Total Lagrange formulation has been adopted to include nonlinear terms in von Kármán sense, which allows to retain the membrane stiffening effect of statically indeterminate problems: the SGUF "fundamental nuclei" for the Ritz solution have been derived for the tangent and secant matrices formulated in the framework of a consistent Newton-Raphson linearisation, which has been solved by means of implicit solution algorithms. Variable kinematics models have been already successfully extended to general geometrical nonlinearity with arbitrarily large displacements in the framework of FEM [99, 125, 151]. All developments adopt to a Total Lagrange formulation and implicit solution methods and concern displacement-based models only. A dedicated post-processing procedure for obtaining the transverse stresses in the deformed configuration (Cauchy stresses) has been proposed in [125]. Future research could be directed towards the use of explicit algorithms, which may alleviate the computational burden of iterative schemes and open the possibility to simulate dynamic processes such as impacts, as well as the extension to geometric nonlinearity of RMVT models. While the extension to material nonlinearity for displacement-based models is quite straight-forward as it follows the very same scheme of conventional FEM, for RMVT-based models attention should be paid to the semi-complementary energy that links the kinematic and static field variables.

A final remark is dedicated to extending the scope of the definition of *pertinence* of a structural model beyond the perspective adopted throughout this memoir. The developed models and associated numerical predictions will in fact have a merely abstract interest unless they are explicitly connected to physical evidence. For being truly predictive, the model should rely on an accurate knowledge of the starting configuration, and it should provide an extrapolation to the final configuration that is connected to physical measurements of the real evolution. Therefore, experimental investigations should be carried out for identifying the model parameters and validating the numerical solutions. On the one hand, high-fidelity models can offer the possibility of accounting for complex physical mechanisms, they may however require time- and cost-intensive experimental investigations, whose results could be heavily affected by inaccuracies and scattering. On the other hand, it is more straight-forward to gather physical evidence for substantiating simple models, whose predictions may however be unacceptably inaccurate when applied to more complex aspects of the design. In this context, a *pertinent* model from the stand-point of engineering applications should be *effectively* connected to the physical measures of the intended structural behaviour.

Bibliography

[1] Abrate, S. and Di Sciuva, M. (2017). Equivalent single layer theories for composite and sandwich structures: A review. *Composite Structures* **179**:482–494 (cit. on p. 10).

[2] Ahn, J.-S., Kim, Y.-W., and Woo, K.-S. (2013). Analysis of circular free edge effect in composite laminates by p-convergent global–local model. *International Journal of Mechanical Sciences* **66**:149–155 (cit. on p. 36).

[3] Ahn, J.-S. and Woo, K.-S. (2014). Interlaminar stress distribution of laminated composites using the mixed-dimensional transition element. *Journal of Composite Materials* **48**:3–20 (cit. on p. 36).

[4] Alessandrini, S. M., Arnold, D. N., Falk, R. S., and Madureira, A. L. (1999). "Derivation and justification of plate models by variational methods". In: Fortin, M., ed. *Plates and Shells*. Vol. 21. CRM Proceedings and Lecture Notes. American Mathematical Society, pp. 1–20 (cit. on p. 15).

[5] Allen, H. G. (1969). *Analysis and Design of Structural Sandwich Panels*. Pergamon Press (cit. on p. 42).

[6] *IEEE Standard on Piezoelectricity* (1987). American National Standard Institute (cit. on p. 23).

[7] Auricchio, F. and Sacco, E. (1999). Partial-mixed formulation and refined models for the analysis of composite laminates within an FSDT. *Composite Structures* **46**:103–113 (cit. on p. 33).

[8] Bathe, K. J. and Dvorkin, E. N. (1985). A four-node plate bending element based on Mindlin/Reissner plate theory and a mixed interpolation. *International Journal for Numerical Methods in Engineering* **21**:367–383 (cit. on p. 32).

[9] Bathe, K.-J. (1996). *Finite Element Procedures*. New Jersey: Prentice-Hall. ISBN: 0-13-301458-4 (cit. on p. 9).

[10] Batoz, J.-L. and Lardeur, P. (1989). A discrete shear triangular nine d.o.f. element for the analysis of thick to very thin plates. *International Journal for Numerical Methods in Engineering* **28**:533–560 (cit. on p. 32).

[11] Ben Dhia, H. and Rateau, G. (2005). The Arlequin method as a flexible engineering design tool. *International Journal for Numerical Methods in Engineering* **62**:1442–1462 (cit. on p. 36).

[12] Benedetti, I. and Milazzo, A. (2017). Advanced models for smart multilayered plates based on Reissner Mixed Variational Theorem. *Composites Part B* **119**:215–229 (cit. on p. 23).

[13] Benjeddou, A. (2000). Advances in piezoelectric finite element modeling of adaptive structural elements: a survey. *Computers & Structures* **76**:347–363 (cit. on p. 35).

[14] Benjeddou, A. (2014). Modal effective electromechanical coupling approximate evaluations and simplified analyses: numerical and experimental assessments. *Acta Mechanica* **225**:2721–2742 (cit. on p. 29).

[15] Berdichevsky, V. L. (1979). Variational-Asymptotic Method of constructing a theory of shells. *Journal of Applied Mathematics and Mechanics (P. M. M.)* **43**:711–736 (cit. on p. 38).

[16] Biscani, F., Giunta, G., Belouettar, S., Carrera, E., and Hu, H. (2011). Variable kinematic beam elements coupled via Arlequin method. *Composite Structures* **93**:697–708 (cit. on p. 36).

[17] Biscani, F., Giunta, G., Belouettar, S., Carrera, E., and Hu, H. (2012). Variable kinematic plate elements coupled via Arlequin method. *International Journal for Numerical Methods in Engineering* **91**:1264–1290 (cit. on p. 36).

[18] Blanco, P. J., Feijóo, R. A., and Urquiza, S. A. (2008). A variational approach for coupling kinematically incompatible structural models. *Computer Methods in Applied Mechanics and Engineering* **197**:1577–1602 (cit. on p. 36).

[19] Botshekanan Dehkordi, M., Cinefra, M., Khalili, S. M. R., and Carrera, E. (2013). Mixed LW/ESL models for the analysis of sandwich plates with composite faces. *Composite Structures* **98**:330–339 (cit. on p. 19).

[20] Bouteiller, P., Bleyer, J., and Sab, K. (2021). Consistent derivation of stress-based generalized models in elastodynamics: Application to layerwise plate models. *International Journal of Solids and Structures* **234-235**:111077 (cit. on p. 15).

[21] Caron, J. F., Diaz Diaz, A., Carreira, R. P., Chabot, A., and Ehrlacher, A. (2006). Multi-particle modelling for the prediction of delamination in multi-layered materials. *Composites Science and Technology* **66**:755–765 (cit. on p. 15).

[22] Carrera, E. (1994). A study on arc-length-type methods and their operation failures illustrated by a simple model. *Computers & Structures* **50**:217–229 (cit. on p. 29).

[23] Carrera, E. (1995). A class of two dimensional theories for multilayered plates analysis. *Atti Accademia delle Scienze di Torino, Memorie Scienze Fisiche* **19-20**:49–87 (cit. on p. 17).

[24] Carrera, E. (1997). C_z^0-Requirements - models for the two dimensional analysis of multilayered structures. *Composite Structures* **37**:373–383 (cit. on p. 10).

[25] Carrera, E. (1998). Evaluation of layerwise mixed theories for laminated plates analysis. *AIAA Journal* **36**:830–839 (cit. on p. 17).

[26] Carrera, E. (2000). A priori vs. a posteriori evaluation of transverse stresses in multilayered orthotropic plates. *Composite Structures* **48**:245–260 (cit. on p. 20).

[27] Carrera, E. (2002). Temperature profile influence on layered plate response considering classical and advanced theories. *AIAA Journal* **40**:1885–1896 (cit. on pp. 23, 35).

[28] Carrera, E. (2002). Theories and Finite Elements for Multilayered, Anisotropic, Composite Plates and Shells. *Archives of Computational Methods in Engineering* **9**:87–140 (cit. on p. 10).

[29] Carrera, E. (2003). Historical review of Zig-Zag theories for multilayered plates and shells. *Applied Mechanics Reviews* **56**:287–308 (cit. on p. 10).

[30] Carrera, E. (2005). Transverse normal strain effects on thermal stress analysis of homogenous and layered plates. *AIAA Journal* **43**:2232–2242 (cit. on p. 23).

[31] Carrera, E. and Brischetto, S. (2008). Analysis of thickness locking in classical, refined and mixed theories for layered shells. *Composite Structures* **85**:83–90 (cit. on p. 35).

[32] Carrera, E., Brischetto, S., and Nali, P. (2008). Variational statements and computational models for multifield problems and multilayered structures. *Mechanics of Advanced Materials and Structures* **15**:182–198 (cit. on p. 23).

[33] Carrera, E., Brischetto, S., and Nali, P. (2011). *Plates and shells for smart structures. Classical and advanced theories for modeling and analysis*. John Wiley & Sons, Ltd. ISBN: 9780470971208 (cit. on p. 17).

[34] Carrera, E., Cinefra, M., Lamberti, A., and Petrolo, M. (2015). Results on best theories for metallic and laminated shells including Layer-Wise models. *Composite Structures* **126**:285–298 (cit. on p. 18).

[35] Carrera, E., Cinefra, M., and Nali, P. (2010). MITC technique extended to variable kinematic multilayered plate elements. *Composite Structures* **92**:1888–1895 (cit. on p. 32).

[36] Carrera, E., Cinefra, M., Petrolo, M., and Zappino, E. (2014). *Finite Element Analysis of Structures through Unified Formulation*. Chichester, UK: John Wiley & Sons, Ltd. ISBN: 978-1-11-994121-7 (cit. on p. 17).

[37] Carrera, E. and Demasi, L. (2002). Classical and advanced multilayered plate elements based upon PVD and RMVT. Part 1: Derivation of finite element matrices. Part 2: Numerical implementations. *International Journal for Numerical Methods in Engineering* **55**:191–231, 253–291 (cit. on pp. 31–33).

[38] Carrera, E. and Fagiano, C. (2007). Mixed Piezoelectric Plate Elements with Continuous Transverse Electric Displacement. *Journal of Mechanics of Materials and Structures* **2** (cit. on p. 23).

[39] Carrera, E., Giunta, G., and Petrolo, M. (2011). *Beam Structures - Classical and Advanced Theories*. John Wiley & Sons, Ltd. ISBN: 978-0-47-097200-7 (cit. on p. 17).

[40] Carrera, E. and Miglioretti, F. (2012). Selection of appropriate multilayered plate theories by using a genetic like algorithm. *Composite Structures* **94**:1175–1186 (cit. on p. 18).

[41] Carrera, E., Miglioretti, F., and Petrolo, M. (2011). Guidelines and recommendations on the use of higher order finite elements for bending analysis of plates analysis. *International Journal for Computational Methods in Engineering Science and Mechanics* **12**:303–324 (cit. on p. 38).

[42] Carrera, E. and Zappino, E. (2017). One-dimensional finite element formulation with node-dependent kinematics. *Computers & Structures* **192**:114–125 (cit. on p. 36).

[43] Chen, W. Q., Cai, J. B., and Ye, G. R. (1993). Exact solutions of cross-ply laminates with bonding imperfections. *AIAA Journal* **41**:2244–2250 (cit. on p. 27).

[44] Chinesta, F., Ammar, A., and Cueto, E. (2010). Recent Advances and New Challenges in the Use of the Proper Generalized Decomposition for Solving Multidimensional Models. *Archives of Computational Methods in Engineering* **17**:327–350 (cit. on p. 38).

[45] Cho, Y. B. and Averill, R. C. (2000). First-order zig-zag sublaminate plate theory and finite element model for laminated composite and sandwich panels. *Composite Structures* **50**:1–15 (cit. on pp. 45, 47).

[46] Chopra, I. (2002). Review of state of art of smart structures and integrated systems. *AIAA Journal* **40**:2145–2187 (cit. on p. 23).

[47] Cinefra, M., Lamberti, A., Zenkour, A. M., and Carrera, E. (2015). Axiomatic/Asymptotic Technique Applied to Refined Theories for Piezoelectric Plates. *Mechanics of Advanced Materials and Structures* **22**:107–124 (cit. on p. 18).

[48] Ciuffreda, A. and Carrera, E. (2005). Bending of composites and sandwich plates subjected to localized lateral loadings: a comparison of various theories. *Composite Structures* **68**:185–202 (cit. on p. 26).

[49] Daniel, I. M., Gdoutos, E. E., Wang, K.-A., and Abot, J. L. (2002). Failure modes of composite sandwich beams. *International Journal of Damage Mechanics* **11**:309–334 (cit. on p. 41).

[50] Davì, G. and Milazzo, A. (1997). Boundary Element Solution for Free Edge Stresses in Composite Laminates. *Journal of Applied Mechanics* **64**:877–884 (cit. on p. 49).

[51] de Borst, R., Remmers, J. J. C., and Needleman, A. (2006). Mesh-independent discrete numerical representations of cohesive-zone models. *Engineering Fracture Mechanics* **73**:160–177 (cit. on p. 56).

[52] Demasi, L. (2008). ∞^3 Hierarchy plate theories for thick and thin composite plates: The generalized unified formulation. *Composite Structures* **84**:256–270 (cit. on pp. 17, 19).

[53] Demasi, L. (2009). ∞^6 Mixed plate theories based on the Generalized Unified Formulation. Part I: Governing equations. *Composite Structures* **87**:1–11 (cit. on pp. 17, 19).

[54] Demasi, L. (2009). ∞^6 Mixed plate theories based on the Generalized Unified Formulation. Part V: Results. *Composite Structures* **88**:1–16 (cit. on p. 20).

[55] Demasi, L. (2012). Partially Zig-Zag advanced higher order shear deformation theories based on the Generalized Unified Formulation. *Composite Structures* **94**:363–375 (cit. on p. 10).

[56] Demasi, L. (2013). Partially Layer Wise advanced Zig Zag and HSDT models based on the Generalized Unified Formulation. *Engineering Structures* **53**:63–91 (cit. on p. 19).

[57] Demasi, L. and Santarpia, E. (2019). Functional reconstitution of Reissner's Mixed Variational Theorem for finite element applications. *AIAA Journal* **57**:3534–3547 (cit. on p. 33).

[58] Dozio, L. and Carrera, E. (2012). Ritz analysis of vibrating rectangular and skew multilayered plates based on advanced variable-kinematics models. *Composite Structures* **94**:2118–2128 (cit. on p. 27).

[59] Echter, R. and Bischoff, M. (2010). Numerical efficiency, locking and unlocking of NURBS finite elements. *Computer Methods in Applied Mechanics and Engineering* **199**:374–382 (cit. on p. 32).

[60] Fagerberg, L. (2005). Effects of anisotropy and multiaxial loading on the wrinkling of sandwich panels. *Journal of Sandwich Structures and Materials* **7**:177–194 (cit. on p. 42).

[61] Feng, W. and Hoa, S. V. (1998). Partial hybrid finite elements for composite laminates. *Finite Elements in Analysis and Design* **30**:365–382 (cit. on p. 36).

[62] Filippi, M., Petrolo, M., Valvano, S., and Carrera, E. (2016). Analysis of laminated composites and sandwich structures by trigonometric, exponential and miscellaneous polynomials and a MITC9 plate element. *Composite Structures* **150**:103–114 (cit. on p. 18).

[63] Fraeijs de Veubeke, B. M. (1965). "Displacement and equilibrium models in the finite element method". In: Zienkiewicz, O. C. and Holister, G. S., eds. *Stress Analysis*. John Wiley & Sons, LTD, pp. 145–197 (cit. on p. 15).

[64] Gallimard, L., Vidal, P., and Polit, O. (2013). Coupling finite element and reliability analysis through proper generalized decomposition model reduction. *International Journal for Numerical Methods in Engineering* **95**:1079–1093 (cit. on p. 38).

[65] Gorgeri, A., Vescovini, R., and Dozio, L. (2019). Analysis of multiple-core sandwich cylindrical shells using a sublaminate formulation. *Composite Structures* **225** (cit. on p. 28).

[66] Groh, R. M. J. and Weaver, P. M. (2015). On displacement-based and mixed-variational equivalent single layer theories for modelling highly heterogeneous laminated beams. *International Journal of Solids and Structures* **59**:147–170 (cit. on p. 20).

[67] Hallett, S. R., Green, B. G., Jiang, W. -G., Cheung, K. H., and Wisnom, M. R. (2009). The open hole tensile test: a challenge for virtual testing of composites. *International Journal of Fracture* **158**:169–181 (cit. on p. 49).

[68] Hildebrand, F. B., Reissner, E., and Thomas, G. B. (1949). *Notes on the foundations of the theory of small displacements of orthotropic shells*. Tech. rep. 1833. National Advisory Committee for Aeronautics (cit. on p. 35).

[69] Hu, H., Belouettar, S., Potier-Ferry, M., and Daya, E. M. (2009). Multi-scale modelling of sandwich structures using the Arlequin method - Part I: Linear modelling. *Finite Elements in Analysis and Design* **45**:37–51 (cit. on p. 37).

[70] Hughes, T. J. R. (1987). *The Finite Element Method*. Prentice-Hall. ISBN: 978-0-48-641181-1 (cit. on p. 32).

[71] Hughes, T. J. R. and Tezduyar, T. (1981). Finite elements based upon Mindlin plate theory with particular reference to the four node bilinear isoparametric element. *Journal of Applied Mechanics* **46**:587–596 (cit. on p. 32).

[72] Infantes, M., Vidal, P., Castro-Triguero, R., Gallimard, L., and Polit, O. (2021). Forced vibration analysis of composite beams with piezoelectric layers based on the variable separation method. *Composite Structures* **273**:114248 (cit. on p. 38).

[73] Jiang, J. P. and Li, D. X. (2007). A new finite element model fro piezothermoelastic composite beam. *Journal of Sound and Vibration* **306**:429–451 (cit. on p. 35).

[74] Latham, C. T., Toledano, A., Murakami, H., and Seible, F. (1988). A shear-deformable two-layer plate element with interlayer slip. *International Journal for Numerical Methods in Engineering* **26**:1769–1789 (cit. on p. 17).

[75] Lebée, A. and Sab, K. (2017). On the generalization of Reissner plate theory to laminated plates, part I: Theory. *Journal of Elasticity* **126**:39–66 (cit. on p. 15).

[76] Lee, S. W. and Pian, T. H. H. (1978). Improvement of plate and shell finite elements by mixed formulations. *AIAA Journal* **16**:29–34 (cit. on p. 33).

[77] Leissa, A. W. (1973). *Vibration of Shells*. Tech. rep. NASA SP-288. NASA (cit. on p. 13).

[78] Leissa, A. W. (2005). The historical bases of the Rayleigh and Ritz methods. *Journal of Sound and Vibration* **287**:961–978 (cit. on p. 26).

[79] Lesieutre, G. A. and Bianchini, E. (1995). Time domain modeling of linear viscoelasticity using anelastic displacement fields. *Journal of Vibration and Acoustics* **117**:424–430 (cit. on p. 28).

[80] Ley, R. P., Lin, W., and Mbanefo, U. (1999). *Facesheet wrinkling in sandwich structures*. Tech. rep. NASA/CR-1999-208994. NASA (cit. on p. 42).

[81] Li, G. and Carrera, E. (2019). On the mitigation of shear locking in laminated plates through $p-$version refinement. *Computers & Structures* **225**:106121 (cit. on p. 32).

[82] Li, G., Carrera, E., Cinefra, M., de Miguel, A. G., Pagani, A., and Zappino, E. (2019). An adaptable refinement approach for shell finite element models based on node-dependent kinematics. *Composite Structures* **210**:1–19 (cit. on p. 36).

[83] Li, M. S. (June 1989). "Higher order laminated composite plate analysis by hybrid finite element method". PhD thesis. Massachusetts Institute of Technology (cit. on p. 50).

[84] Liew, K. M., Pan, Z. Z., and Zhang, L. W. (2019). An overview of layerwise theories for composite laminates and structures: Development, numerical implementation and application. *Composite Structures* **216**:240–259 (cit. on p. 10).

[85] Lo, K. H., Christensen, R. M., and Wu, E. M. (1977). A high-order theory of plate deformation - Part 1: Homogeneous plates - Part 2: Laminated plates. *Journal of Applied Mechanics* **44**:663–668, 669–676 (cit. on p. 22).

[86] Lucking, W. M., Hoa, S. V., and Sankar, T. S. (1984). The Effect of Geometry on Interlaminar Stresses of $[0/90]_s$ Composite Laminates with Circular Holes. *Journal of Composite Materials* **17**:188–198 (cit. on pp. 49, 50).

[87] MacNeal, R. H. (1982). Derivation of element stiffness matrices by assumed strain distributions. *Nuclear Engineering and Design* **70**:3–12 (cit. on p. 32).

[88] Malekzadeh, P. and Karami, G. (2006). Differential quadrature nonlinear analysis of skew composite plates based on FSDT. *Engineering Structures* **28**:1307–1318 (cit. on p. 29).

[89] Malkus, D. S. and Hughes, T. J. R. (1978). Mixed finite element methods – reduced and selective integration techniques. *Computer Methods in Applied Mechanics and Engineering* **15**:63–81 (cit. on p. 15).

[90] Marinkovich, D., Köppe, H., and Gabbert, U. (2009). Aspects of modeling piezoelectric active thin-walled structures. *Journal of Intelligent Material Systems and Structures* **20**:1835–1844 (cit. on p. 35).

[91] Martin, E., Leguillon, D., and Carrère, N. (2010). A twofold strength and toughness criterion for the onset of free-edge shear delamination in angle-ply laminates. *International Journal of Solids and Structures* **47**:1297–1305 (cit. on p. 56).

[92] Matsunaga, H. (1992). The application of a two-dimensional higher-order theory for the analysis of a thick elastic plate. *Computers & Structures* **45**:633–648 (cit. on p. 17).

[93] Moës, N., Dolbow, J., and Belytschko, T. (1999). A finite element method for crack growth without remeshing. *International Journal for Numerical Methods in Engineering* **46**:131–150 (cit. on p. 56).

[94] Moës, N., Stolz, C., Bernard, P.-E., and Chevaugeon, N. (2010). A level set based model for damage growth: The thick level set approach. *International Journal for Numerical Methods in Engineering* **86**:358–380 (cit. on p. 56).

[95] Moleiro, F., Carrera, E., Zappino, E., Li, G., and Cinefra, M. (2020). Layerwise mixed elements with node-dependent kinematics for global–local stress analysis of multilayered plates using high-order Legendre expansions. *Computer Methods in Applied Mechanics and Engineering* **359**:112764 (cit. on p. 36).

[96] Murakami, H. (1986). Laminated composite plate theory with improved in-plane response. *Journal of Applied Mechanics* **53**:661–666 (cit. on pp. 16, 22).

[97] Noor, A. K., Peters, J. M., and Burton, W. S. (1992). Three-dimensional solutions for thermal buckling of multilayered anisotropic plates. *Journal of Engineering Mechanics* **118**:683–701 (cit. on p. 41).

[98] Nouy, A. (2010). A priori model reduction through Proper Generalized Decomposition for solving time-dependent partial differential equations. *Computer Methods in Applied Mechanics and Engineering* **199**:1603–1626 (cit. on p. 37).

[99] Pagani, A. and Carrera, E. (2017). Large-deflection and post-buckling analyses of laminated composite beams by Carrera Unified Formulation. *Composite Structures* **170**:40–52 (cit. on p. 56).

[100] Pagano, N. J. (1970). Exact solutions for rectangular bidirectional composites and sandwich plates. *Journal of Composite Materials* **4**:20–34 (cit. on p. 26).

[101] Pagano, N. J. (1978). Stress fields in composite laminates. *International Journal of Solids and Structures* **14**:385–400 (cit. on p. 15).

[102] Park, K. C., Pramono, E., Stanley, G. M., and Cabiness, H. A. (1989). "The ANS shell elements: earlier developments and recent improvements". In: Noor, A. K., Belytschko, T., and Simo, J. C., eds. *Analytical and Computational Models of Shells*. Vol. 3. CED. ASME, pp. 217–239 (cit. on p. 32).

[103] Peerlings, R. H. J., Geers, M. G. D., de Borst, R., and Brekelmans, W. A. M. (2001). A critical comparison of nonlocal and gradient-enhanced softening continua. *International Journal for Numerical Methods in Engineering* **38**:7723–7746 (cit. on p. 56).

[104] Petrolo, M., Cinefra, M., Lamberti, A., and Carrera, E. (2015). Evaluation of mixed theories for laminated plates through the axiomatic/asymptotic method. *Composites Part B* **76**:260–272 (cit. on p. 18).

[105] Petrolo, M. and Lamberti, A. (2015). Axiomatic/asymptotic analysis of refined layer-wise theories for composite and sandwich plates. *Mechanics of Advanced Materials and Structures* **23**:28–42 (cit. on p. 18).

[106] Pipes, R. B. and Pagano, N. J. (1970). Interlaminar stresses in composite laminates under uniform axial extension. *Journal of Composite Materials* **4**:538–548 (cit. on p. 48).

[107] Polit, O. (1992). “Dévéloppement d’éléments finis de plaque semi-épaisse et de coque semi-épaisse a double courbure”. PhD thesis. ENSAM (cit. on p. 32).

[108] Polit, O. and Touratier, M. (1999). A C^0 eight node finite element based on the semi-thick doubly curved shell theory: comparison with the degenerated approach. *Revue Européenne des Éléments Finis* **8**:111–134 (cit. on p. 32).

[109] Polit, O., Touratier, M., and Lory, P. (1994). A new eight-node quadrilateral shear-bending plate finite element. *International Journal for Numerical Methods in Engineering* **37**:387–411 (cit. on p. 32).

[110] Prathap, G. (1986). Field-consistency - toward a science of constrained multi-strain-field finite element formulations. *Sādhanā* **9**:319–344 (cit. on p. 32).

[111] Prathap, G., Naganarayana, B. P., and Somashekar, B. R. (1988). Field-consistency analysis of the isoparametric eight-noded plate bending element. *Computers & Structures* **29**:857–873 (cit. on p. 32).

[112] Pritz, T. (1996). Analysis of four-parameter fractional derivative model of real solid materials. *Journal of Sound and Vibration* **195**:103–115 (cit. on p. 28).

[113] Reddy, J. N. (2004). *Mechanics of Laminated Composite Plates and Shells: Theory and Analysis*. 2nd. CRC Press (cit. on p. 9).

[114] Reddy, J. N. and Robbins Jr, D. H. (1994). Theories and computational models for composite laminates. *Applied Mechanics Reviews* **47**:147–169 (cit. on p. 10).

[115] Reissner, E. (1944). On the theory of bending of elastic plates. *Journal of Mathematics and Physics* **23**:184–191 (cit. on p. 15).

[116] Reissner, E. (1945). The effect of transverse shear deformation on the bending of elastic plates. *Journal of Applied Mechanics* **12**:A69–A77 (cit. on p. 15).

[117] Reissner, E. (1950). On a variational theorem in elasticity. *Journal of Mathematics and Physics* **29**:90–95 (cit. on p. 15).

[118] Reissner, E. (1984). On a Certain Mixed Variational Theorem and a Proposed Application. *International Journal for Numerical Methods in Engineering* **20**:1366–1368 (cit. on p. 16).

[119] Reissner, E. (1986). On a Mixed Variational Theorem and on a Shear Deformable Plate Theory. *International Journal for Numerical Methods in Engineering* **23**:193–198 (cit. on p. 16).

[120] Robbins Jr, D. H. and Chopra, I. (2006). The effect of laminate kinematic assumptions on the global response of actuated plates. *Journal of Intelligent Material Systems and Structures* **17**:273–299 (cit. on p. 23).

[121] Robbins Jr, D. H. and Reddy, J. N. (1996). Variable kinematic modeling of laminated composite plates. *International Journal for Numerical Methods in Engineering* **39**:2283–2317 (cit. on pp. 17, 36).

[122] Rouleau, L., Deü, J.-F., and Legay, A. (2017). A comparison of model reduction techniques based on modal projection for structures with frequency-dependent damping. *Mechanical Systems and Signal Processing* **90**:110–125 (cit. on p. 28).

[123] Sab, K. and Lebée, A. (2015). *Homogenization of Heterogeneous Thin and Thick Plates*. John Wiley & Sons, Inc. ISBN: 978-1-84-821652-5 (cit. on pp. 11, 15).

[124] Sander, G. (1969). "Application de la méthode des éléments finis à la flexion des plaques". PhD thesis. Université de Liège (cit. on pp. 14, 15).

[125] Santarpia, E. and Demasi, L. (2020). Large displacement models for composites based on Murakami's Zig-Zag Function, Green-Lagrange Strain Tensor, and Generalized Unified Formulation. *Thin-Walled Structures* **150**:106460 (cit. on p. 56).

[126] Saravanos, D. A. and Heyliger, P. R. (1999). Mechanics and computational models for laminated piezoelectric beams, plates and shells. *Applied Mechanics Reviews* **52**:305–319 (cit. on p. 23).

[127] Savoia, M. and Reddy, J. N. (1992). A Variational Approach to Three-Dimensional Elasticity Solutions of Laminated Composite Plates. *Journal of Applied Mechanics* **59**:166–175 (cit. on p. 37).

[128] Sayyad, A. S. and Ghugal, Y. M. (2017). Bending, buckling and free vibration of laminated composite and sandwich beams: A critical review of literature. *Composite Structures* **171**:486–504 (cit. on p. 10).

[129] Simo, J. C. and Hughes, T. J. R. (1986). On the variational foundations of assumed strain methods. *Journal of Applied Mechanics* **53**:51–54 (cit. on p. 15).

[130] Simon, F. (Apr. 2013). *Proposal of reference panels (ST1.1)*. Internal report. GARTEUR AG-20 (cit. on p. 40).

[131] Soldatos, K. P. (1992). A transverse shear deformation theory for homogeneous monoclinic plates. *Acta Mechanica* **94**:195–220 (cit. on p. 17).

[132] Somashekar, B. R., Prathap, G., and Babu, C. Ramesh (1987). A field-consistent, four-noded, laminated, anisotropic plate/shell element. *Computers & Structures* **25**:345–353 (cit. on p. 32).

[133] Sze, K. Y. and Yao, L. Q. (2000). Modelling smart structures with segmented piezoelectric sensors and actuators. *Journal of Sound and Vibration* **235**:495–520 (cit. on p. 35).

[134] Tahani, M. and Nosier, A. (2003). Edge effects of uniformly loaded cross-ply composite laminates. *Materials & Design* **24**:647–658 (cit. on p. 48).

[135] Tessler, A. (1993). An improved plate theory of {1,2}-order for thick composite plates. *International Journal of Solids and Structures* **30**:981–1000 (cit. on p. 35).

[136] Tessler, A., Annett, M. S., and Gendron, G. (2001). A {1,2}-order plate theory accounting for three-dimensional thermoelastic deformations in thick composite and sandwich laminates. *Composite Structures* **52**:67–84 (cit. on p. 35).

[137] Toledano, A. and Murakami, H. (1988). Shear-deformable two-layer plate theory with interlayer slip. *Journal of Engineering Mechanics* **114**:604–623 (cit. on p. 16).

[138] Turon, A., Dávila, C. G., Camanho, P. P., and Costa, J. (2007). An engineering solution for mesh size effects in the simulation of delamination using cohesive zone models. *Engineering Fracture Mechanics* **74**:1665–1682 (cit. on p. 56).

[139] Varelis, D. and Saravanos, D. A. (2008). Non-linear coupled multi-field mechanics and finite element for active multi-stable thermal piezoelectric shells. *International Journal for Numerical Methods in Engineering* **76**:84–107 (cit. on p. 35).

[140] Vel, S. S., Mewer, R. C., and Batra, R. C. (2004). Analytical solution for the cylindrical bending vibration of piezoelectric composite plates. *International Journal of Solids and Structures* **41**:1625–1643 (cit. on p. 43).

[141] Vidal, P., Gallimard, L., and Polit, O. (2014). Explicit solutions for the modeling of laminated composite plates with arbitrary stacking sequences. *Composites Part B* **60**:697–706 (cit. on p. 38).

[142] Vidal, P., Gallimard, L., and Polit, O. (2015). Assessment of variable separation for finite element modeling of free edge effect for composite plates. *Composite Structures* **123**:19–29 (cit. on pp. 38, 50).

[143] Vidal, P., Gallimard, L., and Polit, O. (2016). Thermo-mechanical analysis of laminated composite and sandwich beams based on a variables separation. *Composite Structures* **152**:755–766 (cit. on p. 38).

[144] Vidal, P., Gallimard, L., and Polit, O. (2018). Robust layerwise C^0 finite element approach based on a variable separation method for the modeling of composite and sandwich plates. *Finite Elements in Analysis and Design* **139**:1–13 (cit. on p. 38).

[145] Vidal, P., Gallimard, L., and Polit, O. (2021). Comparison of different degenerated approaches for the modeling of composite shell structures. *Finite Elements in Analysis and Design* **195**:103585 (cit. on p. 38).

[146] Vidal, P. and Polit, O. (2008). A family of sinus finite elements for the analysis of rectangular laminated beams. *Composite Structures* **84**:56–72 (cit. on p. 34).

[147] Vidal, P. and Polit, O. (2009). A refined sine-based finite element with transverse normal deformation for the analysis of laminated beams under thermomechanical loads. *Journal of Mechanics of Materials and Structures* **4**:1127–1155 (cit. on pp. 34, 44, 52).

[148] Vidal, P. and Polit, O. (2013). A refined sinus plate finite element for laminated and sandwich structures under mechanical and thermomechanical loads. *Computer Methods in Applied Mechanics and Engineering* **253**:396–412 (cit. on p. 34).

[149] Vidal, P., Polit, O., and Gallimard, L. (2016). Modeling of composite plates based on Reissner's Mixed Variational Theorem with a variables separation. *Composites Part B* **86**:229–242 (cit. on p. 38).

[150] Washizu, K. (1975). *Variational Methods in Elasticity and Plasticity*. 2nd ed. Pergamon Press. ISBN: 978-0-08-017653-6 (cit. on p. 14).

[151] Wu, B., Pagani, A., Chen, W. Q., and Carrera, E. (2019). Geometrically nonlinear refined shell theories by Carrera Unified Formulation. *Mechanics of Advanced Materials and Structures* **28**:1721–1741 (cit. on p. 56).

[152] Wu, Z. and Wanji, C. (2009). Stress analysis of laminated composite plates with a circular hole according to a single-layer higher-order model. *Composite Structures* **90**:122–129 (cit. on p. 50).

[153] Yu, W. (2005). Mathematical construction of a Reissner-Mindlin plate theory for composite laminates. *International Journal of Solids and Structures* **42**:6680–6699 (cit. on pp. 11, 15).

[154] Yu, W., Hodges, D. H., and Ho, J. C. (2012). Variational asymptotic beam sectional analysis - An updated version. *International Journal of Engineering Sciences* **59**:40–64 (cit. on p. 11).

[155] Yu, W., Hodges, D. H., and Volovoi, V. V. (2002). Asymptotic construction of Reissner-like composite plate theory with accurate strain recovery. *International Journal of Solids and Structures* **39**:5185–5203 (cit. on p. 15).

[156] Zappino, E., Li, G., Pagani, A., and Carrera, E. (2017). Global-local analysis of laminated plates by node-dependent kinematic finite elements with variable ESL/LW capabilities. *Composite Structures* **172**:1–14 (cit. on p. 36).

[157] Zienkiewicz, O. C. and Chan, A. H. C. (1989). “Coupled problems and their numerical solution”. In: Doltsinis, I. S., ed. *Advances in Computational Nonlinear Mechanics*. International Centre for Mechanical Sciences (CISM). Springer Vienna. Chap. 4, pp. 139–176 (cit. on p. 23).

Deuxième partie

Synthèse des Activités

Table des matières

II.1

Formation

Dr.-Ing. Michele D'OTTAVIO (né le 28/02/1975 à Rome, Italie)

Maître de Conférences à l'UFR SITEC et Chercheur au LEME, Université Paris Nanterre

✉ 50, rue de Sèvres – 92410 Ville d'Avray (France) mdottavi@parisnanterre.fr

depuis 2008 **Maître de Conférences**, Membre du Laboratoire Energétique Mécanique Electromagnétisme (LEME), *UFR SITEC, Université Paris Nanterre, Ville d'Avray (France).*

2007–2008 **Post-Doctorat**, *Laboratoire de Mécanique de l'Université Paris X (LMpX), Ville d'Avray (France).*

3/12/2007 Soutenance de la thèse **Advanced Hierarchical Models of Multilayered Plates and Shells Including Mechanical and Electrical Interfaces**

Jury :	Prof. E. Messerschmid	(Uni Stuttgart, Président)
	Prof. B.-H. Kröplin	(Uni Stuttgart, Directeur de thèse)
	Prof. E. Carrera	(Politecnico di Torino, Rapporteur)

2000–2007 **Assistant de recherche (doctorant)**, *Institut für Statik und Dynamik der Luft- und Raumfahrtkonstruktionen (ISD), Universität Stuttgart (Allemagne).*

1994-2000 **Ingénieur Aéronautique et Spatial**, *Politecnico di Torino (Italie).*

1994 **Baccalauréat Scientifique**, *Deutsche Schule Rom (Italie).*

II.2

Activités Scientifiques

II.2.1 Encadrements

II.2.1.1 Thèses de doctorat

Toutes les thèses soutenues entre 2009 et 2017 ont été inscrites au Collège Doctoral Franco-Allemand (CDFA) « Systèmes intelligents en calcul de structure multi-champ » (voir aussi Section II.2.3).

Mehdi Ben Thaïer [PhD1] : thèse en co-tutelle avec E. Carrera (Politecnico di Torino) et soutenue le 23/02/2010 à UPN ; financement par bourse de l'école doctorale ED139.

Titre : *Modélisation numérique de plaques et coques composites à l'aide d'une approche au sens de Reissner-Mindlin enrichie pour les problèmes mécanique et piézo-mécanique*

Composition du jury :

J. Pouget	PR, Sorbonne Université UPMC (Paris)	Président
E. M. Daya	PR, Université de Lorraine (Metz)	Rapporteur
S. De Rosa	PR, Università Federico II (Napoli)	Rapporteur
E. Carrera	PR, Politecnico di Torino	Examinateur
O. Polit	PR, Université Paris Nanterre	Examinateur
P. Vidal	MCF, Université Paris Nanterre	Examinateur
M. D'Ottavio	MCF, Université Paris Nanterre	Examinateur

Taux d'encadrement : 20% (40% P. Vidal, 40% O. Polit)
mon activité d'encadrement a débuté en 2008 à thèse en cours

Production scientifique : [A9⋆]

Résumé : Ce travail a porté sur le développement d'EF pour plaques et coques composites incluant des couches de capteurs et/ou actionneurs piézo-électriques fonctionnant en mode

31. Dans le cadre de l'approche variationnelle aux « déplacements généralisés », une description « couche équivalente » a été utilisée pour la cinématique et une description « couches discrètes » pour le potentiel électrostatique.

La nouveauté des EF proposés réside dans l'utilisation de la loi constitutive 3D pour le couplage piézo-électrique ; pour cela, la cinématique classique basée sur les hypothèses de Reissner-Mindlin (FSDT, 5 paramètres) a été enrichie par 2 paramètres liés aux déplacements hors plan des surfaces inférieure et supérieure de la plaque/coque, ce qui permet de représenter une déformation normale-transverse qui varie linéairement le long de l'épaisseur (pas de verrouillage de Poisson). La variation le long de l'épaisseur de chaque couche du potentiel électrostatique a été décrite par une interpolation linéaire de Lagrange ; l'introduction de couches numériques permet de représenter le champ électrique induit par la déformation de flexion.

Une interpolation à 8 noeuds (CL8) robuste, libre de problèmes numériques tels que les verrouillages en cisaillement transverse et en membrane, a été utilisée pour les DDL mécaniques. Le potentiel électrostatique a été considéré constant sur chaque EF afin de représenter la condition d'équipotentialité aux électrodes. Les EF mécaniques et piézo-électriques de plaque (P7CL8 et P7CL8PZ) et coque (C7CL8 et C7CL8PZ) ont été implantés dans le code EvalEF et validés en comparant avec des nombreuses solutions de référence de la littérature.

Christian Wenzel [PhD2] : thèse en co-tutelle avec E. Carrera (Politecnico di Torino), débutée le 01/12/2009 et soutenue le 07/10/2014 à UPN ; financement interne du LEME.

Titre : *Local FEM analysis of composite beams and plates : Free-edge effect and incompatible kinematics coupling*

Composition du jury :

A. Beakou	PR, Sigma Clermont	Président, Rapporteur
A. Milazzo	PR, Università di Palermo	Rapporteur
E. Carrera	PR, Politecnico di Torino	Examinateur
O. Polit	PR, Université Paris Nanterre	Examinateur
P. Vidal	MCF, Université Paris Nanterre	Examinateur
M. D'Ottavio	MCF, Université Paris Nanterre	Examinateur

Taux d'encadrement : 40% (40% P. Vidal, 20% O. Polit)

Production scientifique : [IC12, IC16, A18⋆, IC11, A16⋆]

Résumé : Les contraintes hors plan, très dangereuses pour les structures composites car stimulant le délaminage, sont souvent confinées dans des régions de taille réduite suite aux forts gradients, voire singularités, introduits par des discontinuités géométriques, matériaux ou de chargement. Ce travail propose une approche numérique alternative à celle adoptée couramment, qui adopte des modèles différents pour les réponses globale (EF 1D poutre ou 2D plaque/coque) et locale (EF solides 3D) : afin de réduire le coût de calcul, l'étude considère l'utilisation d'EF géométriquement 2D (plaque) à cinématique variable pour les adapter à la fois au calcul de la réponse globale et locale.

La première partie de la thèse vise à évaluer la précision du calcul des contraintes hors plan obtenues par des EF plaque à cinématique raffinée. Les évaluations numériques sont conduites en appliquant à des problèmes de bord libre les modèles formulés en CUF et implantés comme UEL dans Abaqus. Un post-traitement spécifique a été programmé permettant d'identifier les paramètres définissant l'intensité de la singularité de la réponse linéaire élastique.

Dans le but de restreindre l'utilisation des modèles raffinés aux régions d'intérêt, la deuxième partie de l'étude propose une technique de couplage d'EF reposant sur des cinématiques différentes. L'approche eXtended Variational Formulation (XVF) a été suivie car elle permet un couplage sans région de recouvrement. A l'aide d'un paramètre scalaire unique, le couplage des modèles peut être adapté à la physique du problème en minimisant les imprécisions liées à la transition entre modèles. L'approche XVF a été implanté dans le code EvalEF pour coupler différents EF poutre, à savoir les modèles classiques (Euler-Bernoulli et Timoshenko) ainsi que les modèles raffinés de la famille Sinus, avec et sans la contrainte normale-transverse. Les résultats obtenus confirment que l’approche permet d’enrichir la précision de la réponse locale à un coût de calcul moindre.

José-Luis Ramirez Arias [PhD3] : thèse débutée le 01/11/2013 et soutenue le 09/12/2016 à UPN ; financement par bourse Colciencias (Colombie) et projet ProMain de la COMUE Paris Lumières.

Titre : *Development of an artificial muscle for a soft robotic hand prosthesis*

Composition du jury :

Y. Aoustin	PR, Ecole Centrale de Nantes	Président
T. Wallmersperger	PR, TU Dresden	Rapporteur
P. Wenger	PR, Ecole Centrale de Nantes	Rapporteur
C. Laschi	PR, Scuola Superiore Sant'Anna (Pisa)	Examinatrice
N. Jouandeau	MCF HDR, Université Paris 8	Examinateur
O. Polit	PR, Université Paris Nanterre	Examinateur
L. Gallimard	PR, Université Paris Nanterre	Examinateur
M. D'Ottavio	MCF, Université Paris Nanterre	Examinateur

Taux d'encadrement : 30% (35% L. Gallimard, 35% O. Polit)
mon activité d'encadrement a porté sur le choix et modélisation des matériaux intelligents

Résumé : Ce travail a porté sur la conception et réalisation d'une prothèse de main basée sur les principes de la robotique douce. Une configuration sous-actionnée a été choisie de par sa masse et consommation d'énergie réduites : un seul moteur électrique pilote le mouvement de chaque doigt, les phalanges étant connectées par des tendons élastiques. Un modèle dynamique a été formulé et un prototype réalisé par impression 3D. Dans le but d'assouplir et stabiliser les prises et de permettre à la main robotique de s'adapter aux objects qu'elle prend, la possibilité a été ensuite explorée d'introduire des matériaux intelligents. De par les besoins en termes de vitesse de réaction et force exercée, la solution a été retenue d'inclure parallèlement aux tendons élastiques, des tendons constitués de fils d'alliage à mémoire de forme (SMA). Actionnés par la chaleur générée électriquement par effet Joule, les tendons en SMA permettent en effet d'adapter la rigidité de chaque doigt. Un modèle thermodynamique 1D du comportement du fil SMA a été choisi qui soit suffisamment précis pour qu'il puisse venir compléter le modèle dynamique du doigt. Une campagne expérimentale a permis d'identifier les paramètres du modèle nécessaire à la simulation de cette nouvelle conception de main robotique. Les résultats préliminaires obtenus avec un deuxième prototype sont très prometteurs.

Thi Huyam Cham Le [PhD4] : thèse débutée le 01/11/2014 et soutenue le 26/06/2019 à UPN ; financement interne du LEME.

Titre : *Modèles théorique et numérique pour la modélisation avancée de structures composites*

Composition du jury :

P. Vannucci	PR, Université Versailles Saint-Quentin	Président
F. Abed-Meraïm	PR, ENSAM Metz	Rapporteur
M. Cinefra	PR, Politecnico di Torino	Rapporteuse
M. Ganapathi	PR, Vellore Institue of Bangalore	Examinateur
O. Polit	PR, Université Paris Nanterre	Examinateur
P. Vidal	PR, Université Paris Nanterre	Examinateur
M. D'Ottavio	MCF, Université Paris Nanterre	Examinateur

Taux d'encadrement : 50% (25% P. Vidal, 25% O. Polit)

Production scientifique : [IC34, IC32, NC6, IC20, A24⋆, NC4, IC25, IC26⋆, B6⋆]

Résumé : Cette thèse a contribué à la formulation d'EF *robustes* pour les modèles raffinés de plaque composites définis dans le cadre de la CUF, comportant des modèles à couche équivalente et à couches discrètes, issus d'une approche déplacement ou mixte (RMVT). La robustesse de l'interpolation EF a été définie en termes d'absence de pathologies numériques liées au rang de la matrice de rigidité, au verrouillage en cisaillement transverse ainsi qu'à la sensibilité à la distortion du maillage. Des interpolations EF à 4 et à 8 noeuds, initialement proposées pour le modèle FSDT et notées respectivement QC4 et CL8, ont été étendues à tous les modèles de la CUF. Des EF *hybrides* ont été considérés à partir des correspondants *mixtes*. Le travail a été entièrement conduit dans le code commercial Abaqus à travers la subroutine UEL. Une campagne numérique étendue a montré les résultats suivants :

(i) Contrairement aux techniques d'intégration réduite/sélective, les interpolations QC4 et CL8 n'introduisent aucun mode de corps rigide supplémentaire ;

(ii) L'intégration selective ne permet pas de résoudre le verrouillage de l'interpolation Serendipity à 8 noeuds ;

(iii) Le verrouillage est associé seulement au terme du cisaillement transverse introduit par la FSDT ;

(iv) Les EF isoparamétriques *hybrides* souffrent des mêmes pathologies que les EF formulés en déplacement ;

(v) Les EF isoparamétriques *mixtes* ne sont pas sujets au verrouillage ;

(vi) Utiliser les interpolations QC4 et CL8 pour les variables cinématiques des EF *mixtes* permet d'augmenter la robustesse vis-à-vis de la distortion du maillage.

Girolamo Di Cara (en cours) : thèse débutée le 01/07/2018, soutenance prévue en 2022 ; financement interne du LEME et aide à la mobilité par le programme PESSOA (projet en collaboration avec A. L. Araújo, Instituto Superior Técnico, Lisboa, Portugal).

Titre provisoire : *Variable kinematics FEM for viscoelastic composite sandwich panels*

Taux d'encadrement : 70% (30% O. Polit)

Production scientifique : [IC33, IC34, IC32, NC6]

Résumé : Le travail porte sur l'analyse par EF de panneaux sandwich avec coeurs en matériaux viscoélastique. Dans l'objectif de minimiser les coûts de calcul, les modèles à cinématique variables formulés dans le cadre de la SGUF sont utilisés. Le formulations aux déplacements et mixtes (RMVT) sont utilisées pour construire les EF en utilisant les interpolations isoparamétriques, QC4 et CL8. Les matériaux viscoélastiques sont modélisés dans le cadre de l'approche aux modules complexes ; la dépendance de la fréquence est modélisée suivant un modèle de Zener à dérivés fractionnaires (4 paramètres) et un modèle de Maxwell généralisé par une expansion en série. Un nouveau code de calcul a été programmé afin d'éviter des limitations des codes commerciales, notamment la solution itérative de problèmes aux valeurs propres complexes. Le développement est validé pour le cas de sandwich élastiques et viscoélastiques.

II.2.1.2 Projets de fin d'études (Master 2) encadrés à l'UFR SITEC

G. Detiège (04/2009 – 08/2009) – M2 UPN, co-encadrement : P. Vidal
Etude du couplage de modèles multiples pour analyse éléments finis [MT1]

F. Kaouach (04/2014 – 08/2014) – M2 UPN, co-encadrement : P. Vidal, O. Polit
Modélisation avancée de matériaux composites sous sollicitations dynamiques [MT2]

E. Pronost (04/2014 – 08/2014) – M2 UPN, co-encadrement : P. Vidal, E. Valot, O. Polit
Etat de l'art sur les assemblages de composites stratifiés et caractérisation quasi-statique d'éprouvettes en carbone-époxyde [MT3]

A.G. Russo (03/2016 – 07/2016) – PFE Uni Palermo, au LEME en mobilité Erasmus
Estensione dei modelli GUF all'analisi di strutture multistrato con interfacce imperfette [MT4]

G. Mantegna (03/2017 – 07/2017) – PFE Uni Palermo, au LEME en mobilité Erasmus
Estensione di un modello di Ritz per pannelli in composito a cinematica variabile in campo non lineare [MT5]

G. Di Cara (03/2018 – 07/2018) – PFE Uni Palermo, au LEME en mobilité Erasmus
Analysis of RMVT based plate models – Development of SGUF Finite Element code for multilayered plate based on mixed formulations [MT6]

S. Bouchatal (04/2019 – 08/2019) – M2 UPN
Evaluation de modèles de plaque dans une approche mixte [MT7]

P.A. Esposito (09/2020 – 03/2021) – PFE Politecnico Milano, co-encadrement : R. Vescovini
Reissner Mixed Variational Theorem for Ritz Sublaminate Generalized Unified Formulation [MT8]

S. Gagliano (03/2021 – 07/2021) – PFE Uni Palermo, au LEME en mobilité Erasmus
Implementation of a frequency-dependent viscoelastic material model for a variable kinematics Finite Element approach for composite plates [MT9]

II.2.1.3 Projets numériques FIPMéca

FIPMéca est une école d'ingénieur en formation continue (voir aussi Section II.3.1) : dans une phase de pré-acquisition, les candidats réalisent un projet numérique sous Matlab. J'ai ainsi proposé des projets afférents à mes activités de recherche :

- ▷ Solutions exactes de plaques composites piézoélectriques (solution de Heyliger).
- ▷ Solutions exactes de plaques sandwich : généralisation de la solution de Pagano à des coeurs avec propriétés d'orthotropie arbitraires (solution de Kardomateas).

- ▷ Extension de la solution de Navier aux chargements développés en série de Fourier : plaques composites (modèles à cinématique variable CUF) et coques composites en flexion cylindrique (solution exacte de Ren).
- ▷ Modèles à cinématique variable (GUF) dans une approche mixte (RMVT) : plaque homogène en flexion cylindrique, plaque composite, et extension au chargement thermique.
- ▷ Visualisation des modes propres d'une matrice de rigidité d'un EF de plaque à cinématique variable.

II.2.2 Projets de recherche

II.2.2.1 Projet Européen FP7 « ANULOID »

Intitulé : Collaborative Project, call ID : FP7-AAT-2012-RTD-L0
"ANULOID – Investigation of novel vertical take-off and landing (VTOL) aircraft concept, designed for operations in urban areas".

Budget : Grant number ACP2-GA-2013-334861-ANULOID : 578 k€ dont 97 k€ pour le LEME.

Durée : 2 ans (avril 2012 – avril 2014)

Partenaires : E. Carrera (Politecnico di Torino, Italie ; coordinateur du projet) ; Z. Pátek (VZLU, République Tchèque) ; C. De Visser (TU Delft, Pays-Bas) ; Z. Janda (FESA, République Tchèque) ; O. Polit (LEME, France).

Resumé du projet : Ce projet a produit une étude de faisabilité d'un avion innovant à décollage vertical pour des missions urbaines. Les contributions du LEME ont concerné la réalisation d'une maquette numérique de l'aéronef ainsi que la réalisation d'un banc d'essais pour mesurer l'efficacité des actionneurs du contrôle du lacet. En tant que responsable scientifique des travaux menés au LEME, j'ai été notamment :

- Encadrant de 3 étudiants :
  - Etudiant en apprentissage : M. M. Bamba Kane (M1 MSCAE, septembre 2013-septembre 2014) : réalisation sous CATIA v5 de la maquette numérique paramétrée de l'ANULOID ; conception et fabrication du banc d'essais pour la mesure du lacet ; mesures expérimentales des vitesses de rotation et du couple produit par les actionneurs.
  - Etudiant en stage : M. E. Usta (M1 MSCAE, mai-juillet 2013) : *Modélisation sous Solidworks Flow Simulation de l'aérodynamique interne de l'ANULOID* : simulation numérique des actionneurs pour le contrôle du lacet.
  - Etudiant en stage : M. A. Lyonnet (M1 MSCAE, mai-juillet 2015) : *Optimisation d'une maquette dans le cadre du projet ANULOID* : conception, fabrication et montage d'une maquette de démonstration avec propulseur intégré.
- Rédacteur des rapports d'activité du LEME pour le consortium :

  Carrera, E., D'Ottavio, M., Petrolo, M., and Polit, O. (Sept. 2013). *Comparison of Anuloid aircraft with other VTOL aircraft*. ANULOID project deliverable D2.6.

  Carrera, E., D'Ottavio, M., Petrolo, M., and Polit, O. (Sept. 2013). *Initial configuration setup*. ANULOID project deliverable D2.1.

Carrera, E., Iuso, G., Petrolo, M., D'Ottavio, M., De Visser, C., Patek, Z., and Janda, Z. (Mar. 2014). *First interim progress report*. ANULOID project deliverable D1.4.

D'Ottavio, M., Polit, O., Janda, Z., Kane, M., and Bolteau, J.-L. (Sept. 2014). *Fuselage with integrated power means, flight control means and other devices*. ANULOID project deliverable D3.4.

D'Ottavio, M., Polit, O., and Kane, M. (Sept. 2014). *Digital mock-up of full-scale Anuloid aircraft*. ANULOID project deliverable D2.2.

- Co-auteur des publications internationales associées au projet [A14, A25]

II.2.2.2 Contrat d'étude industrielle avec Renault SAS

Intitulé : Etat de l'art sur les assemblages de composites stratifiés et caractérisation quasi-statique d'éprouvettes en carbone-époxyde.

Budget : 20 k€.

Durée : 6 mois (avril 2014 – octobre 2014)

Partenaires : O. Polit, E. Valot, P. Vidal, M. D'Ottavio (LEME) ; G. Coma, X. Duhem, J.-F. Lerooy (Renault SAS).

Resumé du projet : Cette étude avait deux objectifs : d'une part, recenser les techniques d'assemblage par fixation des matériaux composites stratifiés ainsi que les moyens de caractériser la tenue de ces assemblages ; d'autre part, la caractérisation d'un matériau composite à fibre longue pour application automobile. Ce projet a été mené en partie dans le cadre du stage de M. Erwann Pronost [MT3] que j'ai co-encadré avec mes collègues du LEME.

II.2.3 Collaborations internationales

Tout au long de mon parcours j'ai eu l'opportunité de travailler avec des collègues dans une dimension internationale, notamment avec l'Italie et l'Allemagne. Les collaborations se sont concrétisées sous différentes formes, de la proposition de projets de recherche européens à l'organisation de réseaux d'échanges pour les étudiants et les chercheurs.

Collaboration avec T. Wallmersperger (TU Dresden) : le Collège Doctoral Franco-Allemand « Systèmes intelligents en calcul de structure multi-champs » La collaboration avec T. Wallmersperger a débuté pendant mon doctorat à Stuttgart et a continué après qu'on ait quitté Stuttgart suite à mon recrutement comme MCF en France et son recrutement en tant que Professeur à la TU Dresden. Nous avons été les promoteurs du Collège Doctoral Franco-Allemand (CDFA) « Systèmes intelligents en calcul de structure multi-champs » au sein de l'Université Franco-Allemande (UFA). Ce dispositif vise à soutenir la recherche par une formation doctorale commune aux établissements partenaires (UPN, TU Dresden et Universität Stuttgart), et il a favorisé les échanges scientifiques sous forme de mobilité des doctorants et d'ateliers de travail communs sur la période 2008 – 2017. Les séminaires communs organisés à intervals réguliers par les établissements partenaire ont permis aux doctorants de parfaire leur technique de présentation des travaux scientifiques devant un publique international. La période de mobilité passée par J.-L. Ramirez Arias au sein de l'équipe de la TU Dresden a conduit à une étude approfondie des propriétés des matériaux intelligents et leur possible utilisation comme muscles artificiels, ainsi qu'à la participation de T. Wallmersperger au jury de thèse de J.-L. Ramirez Arias.

Collaboration avec E. Carrera (Politecnico di Torino) : recherche commune, conférences DeMEASS et projet européen Anuloid J'ai pu travailler avec E. Carrera régulièrement depuis le début de ma thèse, pour laquelle il a été également rapporteur. Les thèses de M. Ben Thaïer et C. Wenzel ont été conduites en co-tutelle avec le Prof. Carrera. J'ai été d'autre part membre de 2 jurys de thèses dirigées par le Prof. Carrera, cf. Section II.2.4.

En 2009 j'ai été invité pour un mois de mobilité au Politecnico di Torino (cadre Erasmus) pour étendre les modèles CUF aux problèmes de flambement (bifurcation), qui a conduit à la publication [A8⋆]. Mme M. Cinefra, membre depuis 2009 du groupe de recherche du Prof. Carrera, a passé 3 semaines au LEME (19 nov – 6 déc 2017) sur un support de Professeur invité du laboratoire : cette collaboration a conduit à la conférence [IC29] et à la publication [A32].

Une collaboration importante s'est réalisée dans le cadre du projet européen Anuloid, comme détaillé Section II.2.2.1.

Le Prof. Carrera a été le promoteur de la série de conférences DeMEASS, auxquelles je participe depuis sa première édition en 2006. J'ai pu contribuer à l'organisation des éditions 2007 (Bad Herrenalb en Allemagne, avec T. Wallmersperger) et 2009 (Vernon en France, avec le LEME) et je suis depuis membre de l'advisory board. Le format de ces conférences – un nombre de participant limité pour avoir une session unique sur 3 jours avec des frais d'inscriptions modérés – favorise les échanges scientifiques et permet aux jeunes doctorants de s'initier aux conférences internationales. Ces conférences ont permis de maintenir un contact régulier avec plusieurs chercheurs européens (S. Belouettar, A. Milazzo, A.L. Araújo...), et ont favorisé la création d'un important réseaux scientifique.

Collaboration avec A. Milazzo (Università di Palermo) : accueil d'étudiants italiens au LEME dans le cadre Erasmus La collaboration avec le Prof. Milazzo a débutée lors des conférences DeMEASS et a conduit notamment à son invitation à rapporter la thèse de C. Wenzel. Nous avons ensuite établi une convention Erasmus pour favoriser la mobilité d'étudiants italiens souhaitant faire une expérience à l'étranger. La mobilité, limitée à la période du PFE et stage, permet d'encourager les étudiants à entreprendre la voie de la recherche. Ainsi, parmi les 3 étudiants italiens que j'ai pu encadrer [MT6, MT5, MT4], G. Di Cara et G. Mantegna sont actuellement en thèse : le premier au LEME depuis l'obtention de son diplôme d'ingénieur, et le deuxième en Italie après avoir passé 2 ans en Angleterre pour parfaire ultérieurement ses connaissances.

Collaboration avec L. Dozio et R. Vescovini (Politecnico di Milano) : développement du code SGUF-Ritz La collaboration a débuté en 2015 lorsque j'ai été rapporteur externe du PFE de C. Punzi, étudiant du Politecnico di Milano encadré par L. Dozio. Dans la même année, L. Dozio a passé un mois comme Professeur invité au LEME. Ensemble nous avons développé une plateforme logicielle combinant l'approche à cinématique variable SGUF avec la méthode de Ritz. R. Vescovini est venu à son tour en Novembre 2016 faire un séminaire au LEME sur la modélisation du post-flambement de plaques et coques composites. Le grand nombre de publications et communications communes (6 articles et 11 conférences entre 2016 et 2021) témoigne de la qualité et régularité de cette collaboration. Sur la période août 2020 – mars 2021 j'ai co-encadré avec R. Vescovini le PFE de P.A. Esposito [MT8], qui a étendu le code SGUF-Ritz aux modèles basés sur l'approche mixtes RMVT pour coques cylindriques ; les mesures sanitaires dictées par la pandémie ont malheureusement interdit la mobilité de l'étudiant sur Paris et l'encadrement a dû se faire entièrement à distance.

Collaboration avec A. L. Araújo (IST, Lisboa) : Programme PESSOA Cette collaboration a débutée en 2019 dans le cadre du programme d'actions intégrées luso-françaises PESSOA, visant à enrichir la thèse de G. Di Cara avec les compétences expérimentales du Prof. Araújo dans le domaine de la caractérisation des structures sandwich viscoélastiques. Le support obtenu par le programme PESSOA a permis G. Di Cara de réaliser

2 séjours de 2 semaines chacun (octobre 2018 et juin 2019) et un séjour de 10 jours (décembre 2022) à l'IST de Lisbonne [IC33, NC6].

Collaboration avec S. Belouettar et G. Giunta (LIST, Luxembourg) Au titre de cette collaboration, débutée dans le cadre des conférences DeMEASS, j'ai pu rédiger les contributions du LEME pour 2 projets européens portés par S. Belouettar, qui n'ont malheureusement pas été financés (FUGRAMMOS en 2008, DURACOMP en 2016). Un échange intéressant a eu lieu en 2010-11 lorsque F. Biscani et C. Wenzel, étudiants en thèse respectivement au LIST et au LEME, ont travaillé sur le couplage de cinématiques incompatibles, le premier en suivant l'approche Arlequin et le deuxième l'approche XVF. En 2013, G. Giunta, ancien étudiant du Prof. Carrera et ensuite recruté au LIST, à passé un mois au LEME en tant que chercheur invité pour travailler sur les modèles raffinées de poutres dans le cadre de la réduction de modèles basée sur la PGD, cf. [IC19, A22, A17].

II.2.4 Activités d'intérêt collectif

Participation à des jurys

- ▷ Examinateur de la thèse de A. Amor : *Actionnement des poutres bistables : modélisations statique et dynamique, optimisations et études expérimentales*, IJLRD, Sorbonne Université (05/06/2020).
- ▷ Rapporteur de la thèse de G. Li : *Variable Kinematic Finite Element Formulations Applied to Multi-layered Structures and Multi-field Problems*, Politecnico di Torino (08/03/2019).
- ▷ Examinateur de la thèse de R. Jamshed : *One-dimensional Advanced Beam Models for Marine Structural Applications*, Politecnico di Torino (27/092017).
- ▷ Rapporteur externe du PFE de C. Punzi : *Variable-kinematics multilayered plate elements for vibro-acoustic analysis of shunted piezoelectric structures*, Politecnico di Milano (2015).

Participation à des comités de sélection extérieurs

- ▷ Université Paul Sabatier, Toulouse (2013).

Participation à des comités scientifiques

- ▷ Membre du Comité Editorial de la section « structures composites » de la revue internationale *Composites Part C* (depuis 2020, date de création de cette revue open access).
- ▷ Membre du Comité Scientifique du symposium international *Design, Modelling and Experiments of Advanced Structures and Systems* (DeMEASS) depuis 2009.
- ▷ Membre du Comité Scientifique des *Journées Nationales sur les Composites 2017* (JNC 20).
- ▷ Membre du Comité Scientifique de l'*International Conference on Mechanics of Nano, Micro and Macro Composite Structures* (ICNMMCS), 2012.
- ▷ Membre du comité d'experts étrangers pour le MIUR (Ministero dell'Istruzione – Università e Ricerca) : expertise d'un projet PRIN (Progetto di Ricerca d'Interesse Nazionale) et d'un projet du Programme Jeunes Chercheurs « Rita Levi Montalcini »
- ▷ Expertise d'une moyenne de 6 articles par an pour des revues internationales à comité de lecture telles que Acta Mechanica, Composites Part B, Composite Structures, International Journal of Solids and Structures, Journal of Applied Mechanics, Journal of Intelligent

Materials Systems and Structures, Mechanics of Advanced Materials and Structures, Thin-Walled Structures …

II.3

Activités Pédagogiques et Administratives

II.3.1 Cadre des activités

Le Pôle Sciences et Technologies (PST) de Ville d'Avray a pris la place de l'ancienne Ecole Technique Aéronautique et se compose aujourd'hui d'un Institut Universitaire Technologique (IUT) et d'une partie de l'Unité de Formation et de Recherche (UFR) *Systèmes Industriels et Techniques de Communication* (SITEC) de l'Université Paris Nanterre.

Depuis 1971, l'IUT propose trois Diplômes Universitaires de Technologie, les DUT Génie Electronique et Informatique Industrielle (GEII), Génie Mécanique et Productique (GMP) et Génie Thermique et Energétique (GTE). A ces formations technologiques de 2 ans se rajoutent des Licences Professionnelles (LP), une année de formation visant la spécialisation de techniciens supérieurs dans un secteur industriel particulier. En cohérence avec l'historique du site, on y retrouve depuis 2004 des LP en lien avec les entreprises des secteurs aéronautique et spatial d'Ile de France, et notamment la LP Structures Aéronautiques et Spatiales (LP SAS), qui prépare aux métiers d'assistants ingénieurs pour les bureaux d'études et de calcul des industries du secteur.

La création de l'UFR SITEC sur le site de Ville d'Avray date de 2000. En 2009, les formations de l'UFR SITEC se sont conformées à la structure Licence-Master-Doctorat, en proposant une Licence *Mathématiques, Informatique et Applications* (MIA) et un Master en Sciences pour l'Ingénieur (SPI), subdivisé en 3 parcours avec les spécialités Electronique, Energétique et Mécanique. Lors du renouvellement des maquettes proposées en 2014, le Master Mécanique de Structures Composites a évolué vers une spécialisation en Aéronautique et Eco-conception (MSCAE). L'UFR SITEC propose depuis 2017 le Cursus de Master en Ingénierie pour l'Aéronautique, les Transports et l'Energétique (CMI-ATE), une formation d'ingénieur sur 5 ans qui s'appuie sur la recherche. L'UFR SITEC propose également une Formation d'Ingénieur en Partenariat spécialité Mécanique (FIPMéca), dont l'objectif est l'évolution en 3 ans des techniciens supérieurs expérimentés vers des fonctions d'ingénieurs diplômés.

II.3.2 Activités d'enseignement

Depuis 2008, je suis intervenu comme enseignant à tous les niveaux des formations proposées au PST de Ville d'Avray, de Bac+1 à Bac+5, en formation initiale, par apprentissage et continue. En moyenne, mon service d'enseignement s'élève à 235 h EqTD/an[1], réparties plutôt équitablement entre les formations Bac+1 – Bac+3 et les formations de niveau supérieur (Master et FIPMéca). En 2019, j'ai débuté une collaboration en tant qu'enseignant vacataire à l'ESTACA (Ecole Supérieure de Technique Aéronautique et de Construction Automobile) : j'interviens dans le programme d'ingénierie « Automotive and Aeronautic Design » (AAD), une formation en anglais dédiée aux étudiants étrangers titulaires d'un *Bachelor Degree* issus des établissements partenaires de l'école.

Les principaux modules dans lesquels je suis intervenu sont indiqués en Table II.3.1 ; pour certains de ces enseignements, j'ai rédigé des supports de cours sous forme de polycopiés ou transparents, dont les plus significatifs sont listés en Table II.3.2. Du fait de l'importance de la professionnalisation dans les formations du PST de Ville d'Avray, j'ai régulièrement assuré le suivi d'étudiants en stage et apprentissage, cf. Table II.3.3 ; cela correspond au suivi d'environ 6 étudiants par an en moyenne.

II.3.3 Activités administratives

En parallèle des enseignements dispensés, j'ai assumé des responsabilités pour le bon fonctionnement des formations du site du PST de Ville d'Avray et je me suis investi dans plusieurs instances locales de l'établissement.

depuis 2021 Directeur adjoint de l'UFR SITEC.

depuis 2021 Responsable du Master Génie Industriel (GI), UFR SITEC.

depuis 2020 Responsable du parcours MSCAE du Master 2 GI, UFR SITEC.

depuis 2018 Membre des Conseils de Perfectionnement du Master GI et du CMI-ATE, UFR SITEC.

2017-21 Membre élu du Conseil de l'UFR SITEC.

depuis 2017 Membre élu du Comité Consultatif Disciplinaire (section 60), UPN.

depuis 2014 Responsable du parcours MSCAE du Master 1 GI, UFR SITEC.

2009-14 Responsable de la LP Structures Aéronautiques et Spatiales, IUT de Ville d'Avray.

[1] 1h CM = 1.5h EqTD, 1h TD = 1h TP = 1h EqTD

Niveau	Matières	Vol. horaire /an CM / TD / TP	Formation	Années
Bac+1	Mécanique du point	6 / 8 / –	L1 MIA	09-14
	Projet initiation ingénierie	– / – / 24	L1 CMI	17-18
Bac+2	Dynamique & Energétique	– / 36 / –	GMP 2	08-09
	Mécanique du Solide 2	18 / – / –	L2 SPI	19-22
	Init. doc. scientifique	– / 15 / –	L2 CMI	18-21
Bac+3	DDS 1	12 / 10 / –	L3 MIA	09-14
	DDS 2	12 / 4 / –	L3 MIA	09-10
		8 / 3 / –	L3 SPI	17-22
	Elasticité	– / 20 / –	LP SAS	09-11
	Calcul de structures	– / 24 / –	LP SAS	08-21
	Méthode des Eléments Finis	12 / 14 / 6	L3 MIA	12-14
		– / – / 40	LP SAS	10-22
	Finite Element Method	23 / – / 24	AAD	18-22
Bac+4	Méthodes numériques	3 / 3 / 4	M1	14-18
	Conception & Vérification	6 / 11 / 4	M1	14-22
Bac+5	Comportement non-linéaire des composites / Structures composites	10 / 14 / 8	M2	09-22
	Modèles raffinés & MEF multiphysique	3 / 3 / 12	M2	09-19
	MEF avancée & multiphysique	3 / 5 / 4	M2	20-22
	Introduction ABAQUS	– / – / 12	M2	09-22
	Architecture des avions	4 / 12 / –	M2	14-20

TABLE II.3.1 – Principaux enseignements dispensés au PST de Ville d'Avray et à l'ESTACA.

Auteurs	Titre du support de cours	pages
M. D'Ottavio	Mécanique du point – Cours & Exercices	48
M. D'Ottavio	Mécanique du solide 2 – Cours & Exercices†	96
M. D'Ottavio	Dimensionnement de Structures 1	55
M. D'Ottavio, O. Polit	Dimensionnement de Structures 2	88
M. D'Ottavio	Méthode des Eléments Finis	40
M. D'Ottavio	Finite Element Method†	80
M. D'Ottavio	Approximations de Solutions par la Méthode de Ritz	24
M. D'Ottavio, L. Gallimard	Critères de Dimensionnement	68
M. D'Ottavio	Comportement Non-Linéaire des Matériaux Composites	95
M. D'Ottavio, P. Vidal, O. Polit	Modèles Raffinés en Calcul de Structures	50
M. D'Ottavio	Architecture des Avions – Conception Aéronautique†	134
M. D'Ottavio	Procédures de résolution de problèmes non-linéaires	37

TABLE II.3.2 – Supports de cours rédigés († support constitué de transparents).

	LPSAS	L3	M1	M2
apprentis	15	–	11	15
stagiaires	15	7	16	8

TABLE II.3.3 – Nombre total d'étudiants par formation suivis en entreprise depuis 2008.

II.4

Production Scientifique

II.4.1 Communications écrites et orales

II.4.1.1 Thèse de doctorat (PhD thesis)

[PhD0] D'Ottavio, M. (2008). "Advanced Hierarchical Models of Multilayered Plates and Shells Including Mechanical and Electrical Interfaces". Universität Stuttgart (cit. on pp. 3, 19, 31, 33).

II.4.1.2 Articles dans revues internationales à comité de lecture (Journal articles)

[A1] Ballhause, D., D'Ottavio, M., Kröplin, B.-H., and Carrera, E. (2005). A unified formulation to assess multilayered theories for piezoelectric plates. *Computers and Structures* **83**:1217–1235. DOI: 10.1016/j.compstruc.2004.09.015 (cit. on p. 23).

[A2] D'Ottavio, M., Ballhause, D., Kröplin, B.-H., and Carrera, E. (2006). Closed-form solutions for the free-vibration problem of multilayered piezoelectric shells. *Computers and Structures* **84**:1506–1518. DOI: 10.1016/j.compstruc.2006.01.030 (cit. on pp. 19, 26).

[A3] D'Ottavio, M., Ballhause, D., Wallmersperger, T., and Kröplin, B.-H. (2006). Considerations on higher-order finite elements for multilayered plates based on a unified formulation. *Computers and Structures* **84**:1222–1235. DOI: 10.1016/j.compstruc.2006.01.025 (cit. on pp. 19, 32).

[A4] D'Ottavio, M. and Kröplin, B.-H. (2006). An extension of Reissner Mixed Variational Theorem to piezoelectric laminates. *Mechanics of Advanced Materials and Structures* **13**:139–150. DOI: 10.1080/15376490500451718 (cit. on pp. 19, 23).

[A5] D'Ottavio, M., Wallmersperger, T., and Kröplin, B.-H. (2008). Classical and advanced models for laminated plates with piezoelectric layers actuated in shear mode. *Mechanics of Advanced Materials and Structures* **15**:167–181. DOI: 10.1080/15376490801907632 (cit. on p. 19).

[A6] Wallmersperger, T., Wittel, F. K., D'Ottavio, M., and Kröplin, B.-H. (2008). Multiscale modeling of polymer gels – Chemo-electric model versus discrete element model. *Mechanics of Advanced Materials and Structures* **15**:228–234. DOI: 10.1080/15376490801907731.

[A7⋆] D'Ottavio, M. and Polit, O. (2009). Sensitivity analysis of thickness assumptions for piezoelectric plate models. *Journal of Intelligent Material Systems and Structures* **20**:1815–1834. DOI: 10.1177/1045389X09343023 (cit. on p. 23).

[A8⋆] D'Ottavio, M. and Carrera, E. (2010). Variable-kinematics approach for linearized buckling analysis of laminated plates and shells. *AIAA Journal* **48**:1987–1996. DOI: 10.2514/1.J050203 (cit. on pp. 19, 26, 88).

[A9⋆] Vidal, P., D'Ottavio, M., Ben Thaïer, M., and Polit, O. (2011). An efficient finite shell element for the static response of piezoelectric laminates. *Journal of Intelligent Material Systems and Structures* **22**:671–690. DOI: 10.1177/1045389X11402863 (cit. on pp. 34, 35, 44, 77).

[A10] Polit, O., Vidal, P., and D'Ottavio, M. (2012). Robust C^0 high-order plate finite element for thin to very thick structures: Mechanical and thermo-mechanical analysis. *International Journal for Numerical Methods in Engineering* **90**:429–451. DOI: 10.1002/nme.3328 (cit. on pp. 34, 35, 47).

[A11⋆] D'Ottavio, M., Vidal, P., Valot, E., and Polit, O. (2013). Assessment of Plate Theories for Free-Edge Effects. *Composites Part B: Engineering* **48**:111–121. DOI: 10.1016/j.compositesb.2012.12.007 (cit. on pp. 47, 48).

[A12⋆] Thai, N. D., D'Ottavio, M., and Caron, J.-F. (2013). Bending analysis of laminated and sandwich plates using a layer-wise stress model. *Composite Structures* **96**:135–142. DOI: 10.1016/j.compstruct.2012.08.032 (cit. on p. 15).

[A13⋆] D'Ottavio, M. and Polit, O. (2014). Linearized global and local buckling analysis of sandwich struts with a refined quasi-3D model. *Acta Mechanica* **225**:81–101. DOI: 10.1007/s00707-014-1169-2 (cit. on pp. 19, 26, 42).

[A14] Petrolo, M., Carrera, E., D'Ottavio, M., De Visser, C., Pàtek, C., and Janda, Z. (2014). On the development of the Anuloid, a disk-shaped VTOL aircraft for urban areas. *Advances in Aircraft and Spacecraft Science* **1**:353–378. DOI: 10.12989/aas.2014.1.3.353 (cit. on p. 87).

[A15] Vidal, P., Polit, O., D'Ottavio, M., and Valot, E. (2014). Assessment of the refined sinus plate finite element: Free edge effect and Meyer-Piening sandwich test. *Finite Elements in Analysis and Design* **92**:60–71. DOI: 10.1016/j.finel.2014.08.004 (cit. on p. 34).

[A16⋆] Wenzel, C., Vidal, P., D'Ottavio, M., and Polit, O. (2014). Coupling of heterogeneous kinematics and Finite Element approximations applied to composite beam structures. *Composite Structures* **116**:177–192. DOI: 10.1016/j.compstruct.2014.04.022 (cit. on pp. 34, 37, 50, 52, 53, 79).

[A17] Polit, O., Gallimard, L., Vidal, P., D'Ottavio, M., Giunta, G., and Belouettar, S. (2015). An analysis of composite beams by means of hierarchical finite elements and a variables separation method. *Computers and Structures* **158**:15–29. DOI: 10.1016/j.compstruc.2015.05.033 (cit. on pp. 38, 90).

[A18⋆] Wenzel, C., D'Ottavio, M., Polit, O., and Vidal, P. (2015). Assessment of free-edge singularities in composite laminates using higher-order plate elements. *Mechanics of Advanced Materials and Structures* **23**:948–959. DOI: 10.1080/15376494.2015.1121526 (cit. on pp. 48, 49, 79).

[A19⋆] D'Ottavio, M. (2016). A Sublaminate Generalized Unified Formulation for the analysis of composite structures and its application to sandwich plates bending. *Composite Structures* **142**:187–199. DOI: 10.1016/j.compstruct.2016.01.087 (cit. on pp. 17, 20, 26).

[A20⋆] D'Ottavio, M., Dozio, L., Vescovini, R., and Polit, O. (2016). Bending analysis of composite laminated and sandwich structures using sublaminate variable-kinematic Ritz models. *Composite Structures* **155**:45–62. DOI: 10.1016/j.compstruct.2016.07.036 (cit. on pp. 27, 46, 47).

[A21⋆] D'Ottavio, M., Polit, O., Ji, W., and Waas, A. M. (2016). Benchmark solutions and assessment of variable kinematics models for global and local buckling of sandwich struts. *Composite Structures* **156**:125–134. DOI: 10.1016/j.compstruct.2016.01.019 (cit. on pp. 26, 42).

[A22] Giunta, G., Belouettar, S., Polit, O., Gallimard, L., Vidal, P., and D'Ottavio, M. (2016). Hierarchical Beam Finite Elements Based Upon a Variables Separation Method. *International Journal of Applied Mechanics* **8**:1650026. DOI: 10.1142/S1758825116500265 (cit. on pp. 38, 90).

[A23⋆] Polit, O., D'Ottavio, M., and Vidal, P. (2016). High-order plate finite elements for smart structure analysis. *Composite Structures* **151**:81–90. DOI: 10.1016/j.compstruct.2016.01.092 (cit. on pp. 34, 35, 44, 45).

[A24⋆] Le, T. H. C., D'Ottavio, M., Vidal, P., and Polit, O. (2017). A new robust quadrilateral four-node variable kinematics plate element for composite structures. *Finite Elements in Analysis and Design* **133**:10–24. DOI: 10.1016/j.finel.2017.05.002 (cit. on pp. 32, 33, 82).

[A25] Petrolo, M., Carrera, E., De Visser, C., D'Ottavio, M., and Polit, O. (2017). Further Results on the Development of a Novel VTOL Aircraft, the Anuloid. Part II: Flight Mechanics. *Advances in Aircraft and Spacecraft Science* **4**:421–436. DOI: 10.12989/aas.2017.4.4.421 (cit. on p. 87).

[A26] Vescovini, R., D'Ottavio, M., Dozio, L., and Polit, O. (2017). Thermal buckling response of laminated and sandwich plates using refined 2-D models. *Composite Structures* **176**:313–328. DOI: 10.1016/j.compstruct.2017.05.021 (cit. on pp. 28, 41).

[A27⋆] D'Ottavio, M., Dozio, L., Vescovini, R., and Polit, O. (2018). The Ritz – Sublaminate Generalized Unified Formulation approach for piezoelectric composite plates. *International Journal of Smart and Nano Materials* **9**:1–22. DOI: 10.1080/19475411.2017.1421275 (cit. on p. 28).

[A28⋆] Vescovini, R., D'Ottavio, M., Dozio, L., and Polit, O. (2018). Buckling and Wrinkling of Anisotropic Sandwich Plates. *International Journal of Engineering Science* **130**:136–156. DOI: 10.1016/j.ijengsci.2018.05.010 (cit. on pp. 42, 43).

[A29⋆] Vescovini, R., Dozio, L., D'Ottavio, M., and Polit, O. (2018). On the Application of the Ritz Method to Free Vibration and Buckling Analysis of Highly Anisotropic Plates. *Composite Structures* **192**:460–474. DOI: 10.1016/j.compstruct.2018.03.017 (cit. on p. 28).

[A30⋆] Loredo, A., D'Ottavio, M., Vidal, P., and Polit, O. (2019). A family of of higher-order single layer plate models meeting C_z^0−requirements for arbitrary laminates. *Composite Structures* **225**. DOI: 10.1016/j.compstruct.2019.111146 (cit. on p. 10).

[A31] Vidal, P., Polit, O., Gallimard, L., and D'Ottavio, M. (2019). Modeling of cylindrical composite shell structures based on the Reissner's Mixed Variational Theorem with a variable separation method. *Advanced Modeling and Simulation in Engineering Sciences* **6**:1–25. DOI: 10.1186/s40323-019-0132-0 (cit. on p. 38).

[A32] Cinefra, M., D'Ottavio, M., Polit, O., and Carrera, E. (2020). Assessment of MITC plate elements based on CUF with respect to distorted meshes. *Composite Structures* **238**:111962. DOI: 10.1016/j.compstruct.2020.111962 (cit. on pp. 32, 88).

[A33⋆] D'Ottavio, M., Krasnobrizha, A., Valot, E., Polit, O., Vescovini, R., and Dozio, L. (2021). Dynamic response of viscoelastic multiple-core sandwich structures. *Journal of Sound and Vibration* **491**:115753. DOI: 10.1016/j.jsv.2020.115753 (cit. on pp. 28, 39).

[A34] Ginot, M., D'Ottavio, M., Polit, O., Bouvet, C., and Castanié, B. (2021). Benchmark of wrinkling formulae and methods for pre-sizing of aircraft lightweight sandwich structures. *Composite Structures*:114387. DOI: 10.1016/j.compstruct.2021.114387 (cit. on p. 42).

II.4.1.3 Contributions à un ouvrage (Book chapters)

[B1] D'Ottavio, M., Wittel, F. K., and Reiser, J. (2006). "Mechanism Based Toolbox: Prototypical Implementation of Tools". In: Busse, G., Kröplin, B.-H., and Wittel, F. K., eds. *Damage and its Evolution in Fiber-Composite Materials: Simulation and Non-Destructive Evaluation*. ISD Verlag, pp. 481–518. ISBN: 3-930683-90-3 (cit. on p. 19).

[B2] Wittel, F. K. and D'Ottavio, M. (2006). "An Open Toolbox for Damage Simulation". In: Busse, G., Kröplin, B.-H., and Wittel, F. K., eds. *Damage and its Evolution in Fiber-Composite Materials: Simulation and Non-Destructive Evaluation*. ISD Verlag, pp. 475–480. ISBN: 3-930683-90-3.

[B3] König, M. and D'Ottavio, M. (2012). "Models for Delamination". In: Blockley, R. and Shyy, W., eds. *Encyclopedia of Aerospace Engineering*. Vol. 3.3. John Wiley: Chichester. Chap. 13. DOI: 10.1002/9780470686652.eae172.pub2 (cit. on p. 56).

[B4⋆] Polit, O., D'Ottavio, M., and Vidal, P. (2014). "Thermal Stress Analysis of Homogeneous and Laminated Shells by Finite Element Method". In: Hetnarski, R. B., ed. *Encyclopedia of Thermal Stresses*. Springer Netherlands. ISBN: 978-94-007-2740-3. DOI: 10.1007/978-94-007-2739-7_290 (cit. on pp. 34, 35).

[B5⋆] D'Ottavio, M. and Polit, O. (2017). "Classical, first order, and advanced theories". In: Abramovich, H., ed. *Stability and Vibration of Thin-Walled Composite Structures*. Composites Science and Engineering. Woodhead Publishing. Chap. 3, pp. 91–140. ISBN: 978-0-08-100410-4. DOI: 10.1016/B978-0-08-100410-4.00003-X (cit. on p. 10).

[B6⋆] Le, T. H. C., D'Ottavio, M., Vidal, P., and Polit, O. (2018). "Robust Displacement and Mixed CUF-Based Four-Node and Eight-Node Quadrilateral Plate Elements". In: Altenbach, H., Carrera, E., and Kulikov, G., eds. *Analysis and Modelling of Advanced Structures and Smart Systems*. Vol. 81. Advanced Structured Materials. Springer, Singapore. Chap. 6, pp. 89–118. ISBN: 978-981-10-6895-9. DOI: 10.1007/978-981-10-6895-9_6 (cit. on pp. 32, 33, 82).

II.4.1.4 Conférences invitées (Invited lectures)

[INV1] D'Ottavio, M., Polit, O., and Vidal, P. (Mar. 2011). "Some benchmarks for assessing models for composite smart structures (Keynote Lecture)". In: *Fourth International Symposium on Design, Modelling and Experiments of Advanced Structures and Smart Systems (DeMEASS IV)*. Urspelt, Luxembourg.

[INV2] D'Ottavio, M. (Sept. 2017). "Classical and advanced models for composite structures based on variable kinematics and a sublaminate approach (Parallel Plenary

Lecture)". In: Ferreira, A. J. M., Larbi, W., Deü, J.-F., Tornabene, F., and Fantuzzi, N., eds. *20th International Conference on Composite Structures*. Paris, France.

[INV3] D'Ottavio, M. (Sept. 2019). "Variable kinematics modeling for advanced composite plates". In: *4th International Workshop on Advanced Dynamics and Model Based Control of Structures and Machines*. Institute of Technical Mechanics, JKU. Linz, Austria.

[INV4] D'Ottavio, M. (June 2021). "Sublaminate variable kinematics approach for the numerical analysis of composite structures (Keynote Lecture)". In: *24th International Conference on Composite Structures*. Porto, Portugal [online conference].

II.4.1.5 Conférences internationales (International conferences)

[IC1] Ballhause, D., D'Ottavio, M., and Carrera, E. (Sept. 2003). "Partially mixed multilayered theories for piezoelectric plates". In: Lipitakis, E. A., ed. *Proceedings of the Sixth Hellenic-European Conference on Computer Mathematics and its Applications (HERCMA 2003)*. Vol. 1. Athens, Hellas: LEA Publishers, pp. 153–163.

[IC2] D'Ottavio, M. and Kröplin, B.-H. (July 2006). "Two-dimensional axiomatic modeling of piezoelectric panels". In: *First International Symposium on Design, Modelling and Experiments of Adaptive Structures and Smart Systems (DeMEASS I)*. Bardonecchia, Italy.

[IC3] D'Ottavio, M., Wittel, F. K., and Kröplin, B.-H. (Sept. 2006). "An Open Toolbox for Damage Simulation". In: *Online Proceedings of CDCM 2006 – Conference on Damage in Composite Materials 2006*. 11 pages. Stuttgart, Germany.

[IC4] D'Ottavio, M. and Polit, O. (Oct. 2007). "Sensititvity analysis of thickness assumptions for axiomatic 2D models of linear elastic and piezoelectric laminates". In: *Second International Symposium on Design, Modelling and Experiments of Adaptive Structures and Smart Systems (DeMEASS II)*. Bad Herrenalb, Germany.

[IC5] D'Ottavio, M., Brischetto, S., and Polit, O. (June 2008). "Hierarchic 2D Models for Piezoelectric Sandwich Shells". In: Schrefler, B. A. and Perego, U., eds. *8th World Congress on Computational Mechanics (WCCM8)*. Venice, Italy.

[IC6] Valot, E., Polit, O., D'Ottavio, M., and Vidal, P. (May 2010). "Influence of 2D finite element models on the 3D stresses evaluation for composite laminates: assessment of anisotropic effects". In: *IV European Conference on Computational Mechanics*. Paris, France.

[IC7] D'Ottavio, M., Polit, O., and Carrera, E. (June 2011). "Assessment of models for the buckling analysis of composite plates and shells". In: Ferreira, A. J. M., ed. *16th International Conference on Composite Structures*. Porto, Portugal.

[IC8] Polit, O., Vidal, P., and D'Ottavio, M. (June 2011). "Seven parameters C^0 F.E. for heterogeneous plate structures". In: Ferreira, A. J. M., ed. *16th International Conference on Composite Structures*. Porto, Portugal.

[IC9] D'Ottavio, M. and Polit, O. (Aug. 2012). "Assessment of Plate Models for Sandwich Bending, Global Buckling and Wrinkling". In: *10th International Conference on Sandwich Structures*. Nantes, France (cit. on p. 19).

[IC10] D'Ottavio, M. and Polit, O. (June 2012). "Quasi-3D solutions for sandwich buckling and wrinkling". In: Ferreira, A. J. M. and Carrera, E., eds. *International Conference on Mechanics of Nano, Micro and Macro Composite Structures*. Turin, Italy (cit. on p. 19).

[IC11] Wenzel, C., D'Ottavio, M., Vidal, P., and Polit, O. (Oct. 2012). "Heterogeneous kinematical analysis of beam structures via the Extended Variational Formulation". In: *Fifth International Symposium on Design, Modelling and Experiments of Advanced Structures and Smart Systems (DeMEASS V)*. Ulrichsberg, Austria (cit. on p. 79).

[IC12] D'Ottavio, M., Wenzel, C., Vidal, P., Valot, E., and Polit, O. (June 2013). "Free-edge stress fields in composite laminates: assessment of finite plate elements". In: Ferreira, A. J. M., ed. *17th International Conference on Composite Structures*. Porto, Portugal (cit. on p. 79).

[IC13] Polit, O., D'Ottavio, M., and Vidal, P. (June 2013). "Robust shell finite element for piezoelectric analysis". In: Carrera, E., Cinefra, M., Miglioretti, F., and Petrolo, M., eds. *Proceedings of the 6th ECCOMAS Thematic Conference on Smart Structures and Materials (SMART2013)*. Torino, Italy.

[IC14] D'Ottavio, M. and Polit, O. (Sept. 2014). "Refined 2D models for buckling and vibration of cylindrical sandwich shells". In: *International Symposium on Dynamic Response and Failure of Composite Materials*. Ischia, Italy (cit. on p. 19).

[IC15] Polit, O., D'Ottavio, M., and Vidal, P. (May 2014). "Robust plate/shell finite element with Zig-Zag function for piezoelectric analysis". In: *Sixth International Symposium on Design, Modelling and Experiments of Advanced Structures and Smart Systems (DeMEASS VI)*. Ede, The Netherlands.

[IC16] Wenzel, C., D'Ottavio, M., Polit, O., and Vidal, P. (May 2014). "Plate finite elements applied to the free-edge singularity in bending and extension". In: *Sixth International Symposium on Design, Modelling and Experiments of Advanced Structures and Smart Systems (DeMEASS VI)*. Ede, The Netherlands (cit. on p. 79).

[IC17] D'Ottavio, M. and Polit, O. (Nov. 2015). "A Generalized Unified Formulation for vibration analysis of prestressed laminated panels". In: *Proceedings of the 23rd Conference of the Italian Association of Aeronautics and Astronautics (AIDAA 2015)*. Torino, Italy (cit. on p. 26).

[IC18] D'Ottavio, M. and Polit, O. (June 2015). "Variable kinematics models for buckling and vibration of multilayered panels". In: Ferreira, A. J. M., ed. *18th International Conference on Composite Structures*. Lisbon, Portugal.

[IC19] Giunta, G., Belouettar, S., Polit, O., Gallimard, L., Vidal, P., and D'Ottavio, M. (Jan. 2015). "Analysis of three-dimensional beams via hierarchical finite elements and a proper generalized decomposition". In: *4th African Conference on Computational Mechanics*. Marrakech, Morocco (cit. on p. 90).

[IC20] Le, T. H. C., D'Ottavio, M., Vidal, P., and Polit, O. (Oct. 2015). "Numerical assessment of a new QC4 finite element for variable-kinematics composite plates". In: *Seventh International Symposium on Design, Modelling and Experiments of Advanced Structures and Smart Systems (DeMEASS VII)*. Radebeul, Germany (cit. on p. 82).

[IC21] Polit, O., D'Ottavio, M., and Vidal, P. (June 2015). "High-order plate/shell FE for piezoelectric analysis: the bimorph configuration". In: Araújo, A. L. and Mota Soares, C. A., eds. *Proceedings of the 7th ECCOMAS Thematic Conference on Smart Structures and Materials (SMART2015)*. Ponte Delgada, Portugal. ISBN: 978-989-96276-8-0.

[IC22] D'Ottavio, M., Dozio, L., Vescovini, R., and Polit, O. (Sept. 2016). "An analysis tool for composite plates based on the Ritz method and a sublaminate generalized unified formulation". In: Ferreira, A. J. M., ed. *19th International Conference on Composite Structures*. Porto, Portugal.

[IC23] D'Ottavio, M., Dozio, L., Vescovini, R., and Polit, O. (Sept. 2016). "Dynamic analysis of multilayered plates with viscoelastic layers using a sublaminate generalized unified formulation". In: Ferreira, A. J. M., ed. *19th International Conference on Composite Structures*. Porto, Portugal.

[IC24] D'Ottavio, M., Dozio, L., Vescovini, R., and Polit, O. (June 2017). "Extension to piezoelectric plates of the Ritz - Sublaminate Generalized Unified Formulation approach". In: Güemes, A., Benjeddou, A., Rodellar, J., and Leng, J., eds. *Proceedings of the 8th ECCOMAS Thematic Conference on Smart Structures and Materials (SMART 2017)*. Madrid, Spain, pp. 1089–1100. ISBN: 978-84-946909-3-8 (cit. on p. 43).

[IC25] Le, T. H. C., D'Ottavio, M., Vidal, P., and Polit, O. (May 2017). "Robust CUF-based four-node and eight-node quadrilateral plate elements". In: *Eight International Symposium on Design, Modelling and Experiments of Advanced Structures and Smart Systems (DeMEASS VIII)*. Moscow, Russia (cit. on p. 82).

[IC26] Le, T. H. C., D'Ottavio, M., Vidal, P., and Polit, O. (Sept. 2017). "Robust four-node and eight-node quadrilateral elements for variable-kinematics analysis of composite plates". In: Ferreira, A. J. M., Larbi, W., Deü, J.-F., Tornabene, F., and Fantuzzi, N., eds. *20th International Conference on Composite Structures*. Paris, France (cit. on p. 82).

[IC27] Loredo, A., Polit, O., D'Ottavio, M., and Vidal, P. (Sept. 2017). "A procedure to fulfill C_z^0-requirements for higher-order plate models and general lamination sequences". In: Ferreira, A. J. M., Larbi, W., Deü, J.-F., Tornabene, F., and Fantuzzi, N., eds. *20th International Conference on Composite Structures*. Paris, France.

[IC28] Vescovini, R., Dozio, L., D'Ottavio, M., and Polit, O. (Sept. 2017). "An investigation over the effects of anisotropy on the convergence of the Ritz method for thin and thick composite plates". In: Ferreira, A. J. M., Larbi, W., Deü, J.-F., Tornabene, F., and Fantuzzi, N., eds. *20th International Conference on Composite Structures*. Paris, France.

[IC29] Cinefra, M., D'Ottavio, M., Polit, O., and Carrera, E. (July 2018). "On the robustness of MITC9 shell elements based on CUF". In: *10th European Solid Mechanics Conference*. Bologna, Italy (cit. on p. 88).

[IC30] D'Ottavio, M., Mantegna, G., and Polit, O. (Oct. 2018). "Geometrically Non-Linear Variable Kinematics Plate Models". In: *Nineth International Symposium on Design, Modelling and Experiments of Advanced Structures and Smart Systems (DeMEASS VIII)*. Sesimbra, Portugal (cit. on p. 29).

[IC31] Dozio, L., Vescovini, R., and D'Ottavio, M. (June 2018). "Sublaminate variable-kinematic models for vibroacoustic analysis of composite trim panels". In: *First International Conference on Mechanics of Advanced Materials and Structures (IC-MAMS 2018)*. Torino, Italy (cit. on p. 28).

[IC32] Di Cara, G., Le, T. H. C., D'Ottavio, M., and Polit, O. (July 2019). "Finite Plate Elements with Variable Kinematics based on Sublaminate Generalized Unified Formulation". In: Ferreira, A. J. M., ed. *5th International Conference on Mechanics of Composites*. Lisbon, Portugal (cit. on pp. 34, 82, 83).

[IC33] Araújo, A. L., Madeira, J. F. A., Di Cara, G., D'Ottavio, M., and Polit, O. (Sept. 2020). "Optimal design of composite laminated structures for noise attenuation using SGUF and DMS". In: Ferreira, A. J. M., Fantuzzi, N., and Bacciocchi, M., eds. *23th International Conference on Composite Structures & 6th International Conference on Mechanics of Composites*. Porto, Portugal [online conference] (cit. on pp. 28, 83, 90).

[IC34] Di Cara, G., D'Ottavio, M., Le, T. H. C., and Polit, O. (Sept. 2021). "FE implementation of Sublaminate Generalised Unified Formulation models for sandwich plates". In: *7th International Conference on Mechanics of Composites*. Porto, Portugal [online conference] (cit. on pp. 34, 82, 83).

II.4.1.6 Conférences nationales (National conferences)

[NC1] D'Ottavio, M. and Polit, O. (May 2009). "Une hiérarchie de modèles raffinés pour plaques composites: application aux effets de bord". In: *9e Colloque National en Calcul des Structures*. Giens, France.

[NC2] Thai, N. D., Caron, J.-F., and D'Ottavio, M. (June 2011). "Benchmark d'un modèle pour sandwiches et multicouches, de type layerwise en contrainte". In: *17èmes Journées Nationales sur les Composites (JNC17)*. AMAC. Poitiers, France.

[NC3] Vidal, P., D'Ottavio, M., and Polit, O. (May 2011). "Un E.F. C^0 basé sur un modèle raffiné pour l'analyse multyphysique de plaques hétérogènes". In: *10e Colloque National en Calcul des Structures*. Giens, France.

[NC4] Le, T. H. C., D'Ottavio, M., Vidal, P., and Polit, O. (June 2017). "Modèles théoriques et numériques pour la modélisation avancée de plaques compositeslisation avancée de plaques composites". In: *Comptes Rendus des JNC 20*. Champs sur Marne, France (cit. on p. 82).

[NC5] Vidal, P., Gallimard, L., D'Ottavio, M., and Polit, O. (May 2017). "Une famille de modèles raffinés pour l'analyse multiphysique de plaques hétérogènes basée sur un E.F. C^0". In: *13ème Colloque National en Calcul des Structures*. Giens, France.

[NC6] Di Cara, G., Le, T. H. C., D'Ottavio, M., Polit, O., and Araújo, A. L. (June 2021). "Éléments de plaque pour structures sandwich avec âme viscoélastique basés sur la Sublaminate Generalized Unified Formulation". In: *JNC 22: Journées Nationales sur les Composites 2021*. Conférence Virtuelle (cit. on pp. 34, 82, 83, 90).

II.4.1.7 Rapports internes (Internal reports)

[Rep1] D'Ottavio, M. (2018). *Some partially mixed variational formulations for laminates*. LEME, Université Paris Nanterre (cit. on p. 56).

[Rep2] D'Ottavio, M. (2020). *Algebraic analysis of RMVT based plate models*. LEME, Université Paris Nanterre (cit. on p. 56).

II.4.2 Mémoires issus des encadrements

II.4.2.1 Thèses de doctorat (PhD theses)

[PhD1] Ben Thaïer, M. (2010). "Modélisation numérique de plaques et coques composites à l'aide d'une approche au sens de Reissner-Mindlin enrichie pour les problèmes mécanique et piézo-mécanique". Université Paris Nanterre (cit. on pp. 35, 77).

[PhD2] Wenzel, C. (2014). "Local FEM analysis of composite beams and plates: Free-edge effect and incompatible kinematics coupling". Université Paris Nanterre (cit. on pp. 19, 34, 36, 47, 48, 52, 79).

[PhD3] Ramirez Arias, J.-L. (2016). "Development of an artificial muscle for a soft robotic hand prosthesis". Université Paris Nanterre (cit. on p. 81).

[PhD4] Le, T. H. C. (2019). "Modèles théorique et numérique pour la modélisation avancée de structures composites". Université Paris Nanterre (cit. on pp. 19, 32–34, 47, 49, 82).

II.4.2.2 Mémoires de fin d'études (Master theses)

[MT1] Detiège, G. (2009). "Etude du couplage de modèles multiples pour analyse éléments finis". Université Paris Nanterre (cit. on p. 84).

[MT2] Kaouach, F. (2014). "Modélisation avancée de matériaux composites sous sollicitations dynamiques". Université Paris Nanterre (cit. on p. 84).

[MT3] Pronost, E. (2014). "Etat de l'art sur les assemblages de composites stratifiés et caractérisation quasi-statique d'éprouvettes en carbone-époxyde". Université Paris Nanterre (cit. on pp. 84, 87).

[MT4] Russo, A. G. (2016). "Estensione dei modelli GUF all'analisi di strutture multistrato con interfacce imperfette". Università degli Studi di Palermo - Scuola Politecnica (cit. on pp. 26, 84, 89).

[MT5] Mantegna, G. (2017). "Estensione di un modello di Ritz per pannelli in composito a cinematica variabile in campo non lineare". Università degli Studi di Palermo - Scuola Politecnica (cit. on pp. 29, 84, 89).

[MT6] Di Cara, G. (2018). "Analysis of RMVT based plate models – Development of SGUF Finite Element code for multilayered platebased on mixed formulations". Università degli Studi di Palermo - Scuola Politecnica (cit. on pp. 34, 84, 89).

[MT7] Bouchatal, S. (2019). "Evaluation de modèles de plaque dans une approche mixte". Université Paris Nanterre (cit. on pp. 56, 84).

[MT8] Esposito, P. A. (2021). "Reissner Mixed Variational Theorem for Ritz Sublaminate Generalized Unified Formulation". Politecnico di Milano (cit. on pp. 28–30, 84, 89).

[MT9] Gagliano, S. (2021). "Implementation of a frequency-dependent viscoelastic material model for a variable kinematics Finite Element approach for composite plates". Università degli Studi di Palermo - Scuola Politecnica (cit. on pp. 34, 84).